

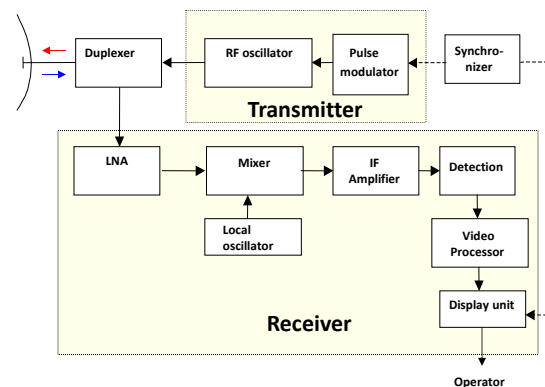
Chapter 2: Pulse Radar

Summary

- **Basic operation and components:** Signal waveform; Pulse repetition frequency; Distance to target ambiguity.
- **Detection:** Minimum detectable signal; Noise at detection characterization; Probability of detection and false alarm; Mean time between false alarms; Pulse integration; Matched filter detection and correlators.
- **Radar Cross Section:** Swerling fluctuations models; Probability of detection under RCS fluctuations
- **Propagation effects:** Ground reflection, atmosphere, obstacles, scattering and dispersion.

2.1 Basic Operation and Components

Simplified block diagram of a pulse radar



2.1 Basic Operation and Components

Notes:

- The transmitter is mainly a radio-frequency (RF) oscillator, with the appropriate power at the required frequency band, and a pulse modulator.
- The receiver is almost always of the super-heterodyne type; A second detector comes after the frequency conversion process.
- The same antenna may be used in a transmitter and receiver shared base by using a duplexer.
- After detection, the raw video (that is, non-processed video) is obtained, containing all the possible information to be extracted.

2.1 Basic Operation and Components

Notes:

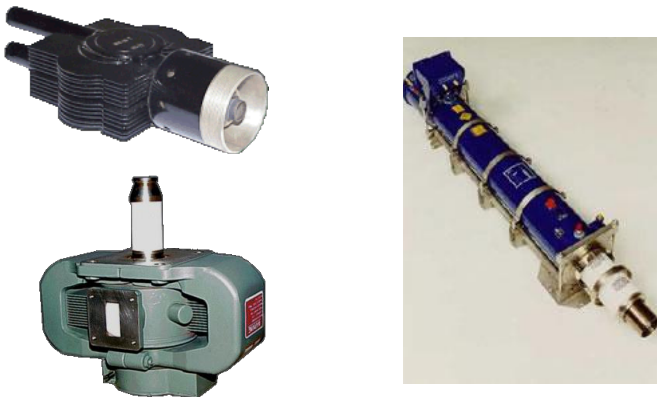
- The less sophisticated radars simply use a video amplifier to visualize the returned signals on a monitor, in many cases a Plane Position Indicator (PPI), or a common computer like screen.
- Post-processing of the raw video includes digitalization, apply detection algorithms, and data compress algorithms, before the information is transmitted to a remote location.
- Several pulse radar techniques have not been included in the previous schematic, such as MTI Processing, which is a must for filtering clutter e extracting Doppler information, and Pulse Compression, that allows the advantages of short duration pulses at reception, and longer duration at the transmitter
- When MTI is used, phase sensitivity detectors are needed, and a number of returned pulses are to be stored in a memory to allow an effective filtering of the clutter and also process the Doppler information.

2.2 Transmitters for Pulse Radars

Transmitters, that generate short duration pulses with high energy content, should:

- Generate the needed RF power;
- Have enough bandwidth;
- Be stable;
- Have the capacity of handling the adequate waveforms;
- Be efficient, reliable, easy maintenance, adequate lifetime, moderate cost.

2.2 Transmitters for Pulse Radars



2.2 Transmitters for pulse Radars

The transmitter can be:

- A **Power Oscillator Transmitter (POT)**: A self-excited oscillator (ex: magnetron), that must have a high-power DC source to generate pulses in a separate unit, the modulator;
- A **Power-Amplifier Transmitter (PAT)**: A master oscillator coupled to a power amplifier (**MOPA**) (Ex: Klystron, Travelling Wave Tube, Solid-state Amplifier);
 - A special case of PAT is used in active antennas, when they have dedicated amplifiers, either alone or in groups.

2.2 Transmitters for Pulse Radars

Technology		Maximum frequency	Peak/ Average Power	Typical gain	Typical bandwidth
POT	Magnetron	95 GHz	1 MW / 500 W	-	Fixed...10%
	Impatt diode	140 GHz	30 W / 10 W	-	Fixed...5%
	Extended interaction oscillator (EIO)	220 GHz	1 kW / 10 W	-	0.2% (elec.) 4% (mech.)
PAT	Helix TWT	95 GHz	4 kW / 200 W	40...60dB	Octave/ multioctave
	Ring-loop TWT	18 GHz	8 kW / 400 W	40...60dB	5...15%
	Coupled-cavity TWT	95 GHz	100 kW / 25 kW	40...60dB	5...15%
	Extended interaction Klystron (EIK)	280 GHz	1 kW / 10 W	40...50dB	0.5...1%
	Klystron	35 GHz	50 kW / 5 kW	30...60dB	0.1...2% (inst.) 1...10% (mech.)
	Crossed-Field amplifier (CFA)	18 GHz	500 kW / 1 kW	10...20dB	5...15%
	Solid state Silicon BJT	5 GHz	300 W / 30 W	5...10dB	10...25%
	GaAs FET	30 GHz	15W / 5 W	5...10dB	5...20%

2.3 Radar Receiver

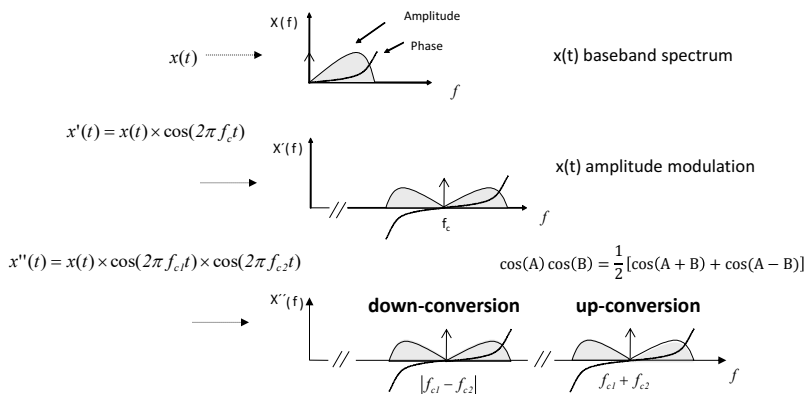
Receivers are usually of the heterodyne type, that make use of frequency conversion:

- The RF pulses go first through an RF amplifier, with a large bandwidth, and then through a low noise amplifier (LNA);
- Pulses go then under a frequency conversion process to an intermediate frequency, which is achieved after mixing with a local oscillator, followed by band-pass filter;
- The intermediate frequency is the difference between the center frequency of the incoming pulses and the local oscillator (LO) frequency;
- At the mixer, sum and difference frequencies are produced; For each frequency there exists an image frequency that also converts to the intermediate frequency:

$$f_{image} = f_{signal} \pm 2f_{IF}$$

2.3 Radar Receiver

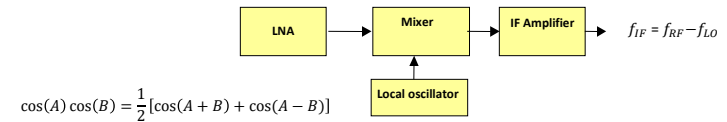
Frequency Conversion



The resulting spectrum is obtained by a shift and symmetric side-bands appear around f_c ;

2.3 Radar Receiver

Super-heterodyne receiver

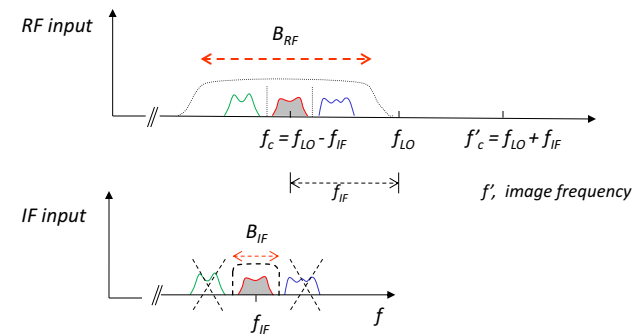


- The intermediate frequency (IF) is the frequency difference between the input signal frequency and the fixed local oscillator (LO) frequency;
- The mixer produces the sum and difference frequencies; in the detection process, if an image frequency is present it also converts to the intermediate frequency

$$f_{image} = f_{signal} \pm 2f_{IF}$$

2.3 Radar Receiver

Heterodyne receiver

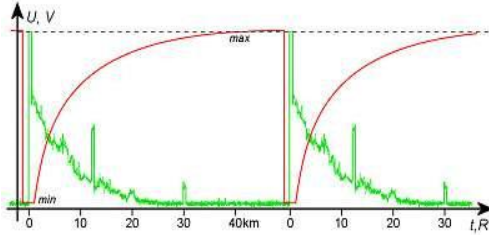


Note: This receiver type calls for a process of image frequency suppression, which is usually done by carefully designing the RF stage with a bandwidth less than $2f_{IF}$

2.3 Radar Receiver

Sensitivity Time Control (STC)

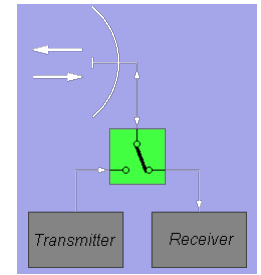
The high dynamic range requires a gain adjustment. Gain should be smaller for near targets and higher for distant targets.



STC circuits allow gain increase as time passes (swept gain attenuator), by means of voltage control at the IF stage, thus controlling its gain.

When pulses end the voltage control bias re-starts the cycle. Ideally, gain variation should compensate the returned pulse power varying as R^4 .

2.4 Duplexers

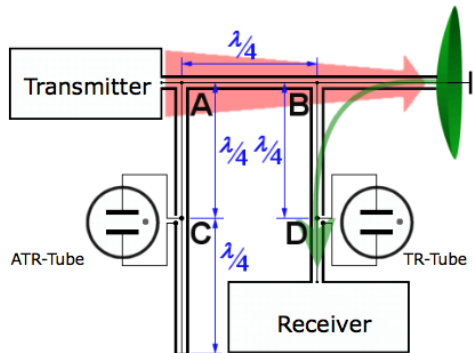


Can be built using several technologies:

- Ferrite circulators (hybrid-ring duplexer);
- Coaxial or waveguide resonators;
- TR e ATR valves;
- PIN diodes.

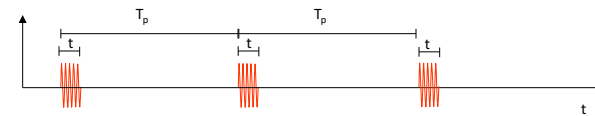
2.4 Duplexers

Stub duplexer with TR and ATR valves



2.5 Pulse Radar Waveform

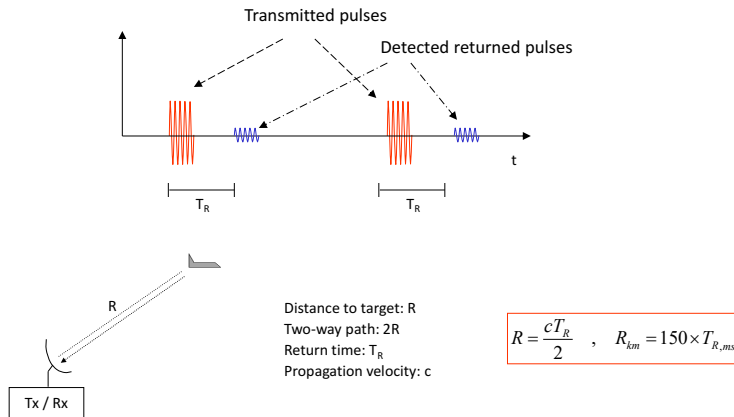
Periodic sequence of modulated RF pulses



- Pulse repetition time: T_p
- Pulse repetition frequency: $f_p = 1/T_p$ (prf)
- Pulse duration: τ (also known as pulse width pw)
- Spectral bandwidth: $\sim 2/\tau$
- Duty cycle: $\tau/T_p = f_p \tau$

2.5 Pulse Radar Waveform

Distance to Target

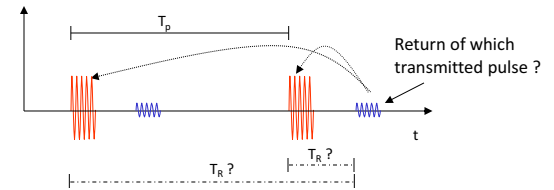


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2.5 Pulse Radar Waveform

Range ambiguity due to periodicity



When the two-way propagation time is higher than the time between two pulses in a row, ambiguity occurs in the distance determination.

MUR – maximum unambiguous range

$$MUR = R_{u,amb} = \frac{cT_p}{2} = \frac{c}{2f_p}$$

Note

$T_p \equiv prt$ (pulse repetition time)

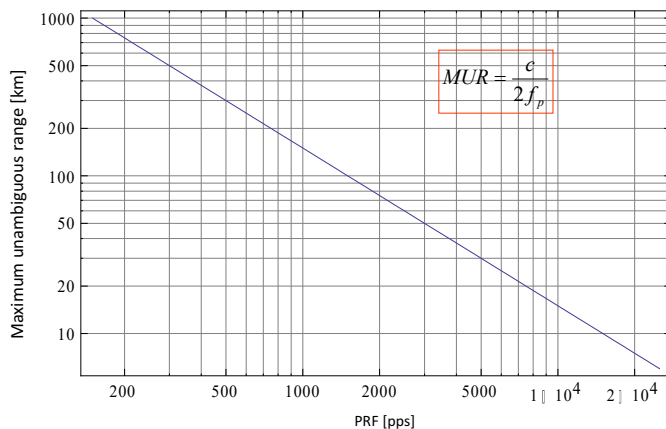
$f_p \equiv prf$ (pulse repetition frequency)

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2.5 Pulse Radar Waveform

MUR vs PRF



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2.5 Pulse Radar Waveform

Strategies to resolve range ambiguity

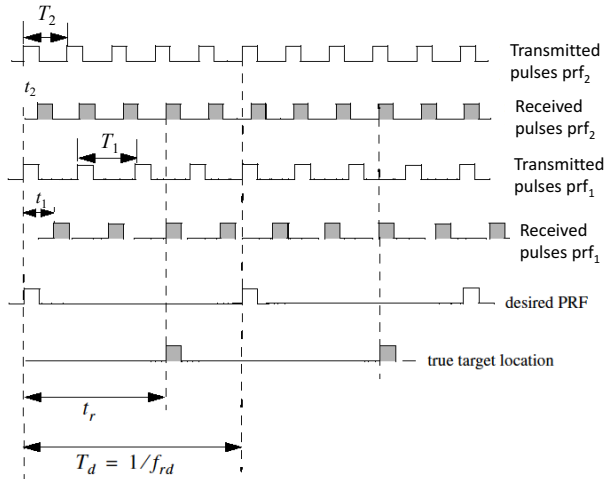
- Consider an unambiguous range goal R_u to which corresponds a repetition frequency prf_{rd} . Choose prf_1 and prf_2 relatively prime, with respect to each other. As an example, lets take $prf_1 = N prf_r$ and $prf_2 = (N+1) prf_r$, where N is an integer.
- Within one period of the desired repetition time interval, the returns, corresponding to the two $prfs$, coincide only at one location, which is the true unambiguous target position.
- The time delay T_d establishes the desired unambiguous range $c/2prf_{rd}$. The time delays t_1 and t_2 correspond to the time between the transmit of a pulse on each prf and receipt of a target return.
- Assume that M_1 (M_2) is the number of time intervals T_1 (T_2), between the transmission and reception, of a true target return.

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2.5 Pulse Radar Waveform

Strategies to resolve range ambiguity



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2.5 Pulse Radar Waveform

Strategies to resolve range ambiguity

- Then, over the interval $[0, T_d]$ the only possible cases are: Either $M_1 = M_2 = M$ or $M_1 + 1 = M_2$.

- Let's consider the case $t_1 < t_2$:

In this case $t_1 + \frac{M}{prf_1} = t_2 + \frac{M}{prf_2}$, thus we get $M = \frac{t_2 - t_1}{T_1 - T_2}$ and $t_r = MT_1 + t_1$ (also $t_r = MT_2 + t_2$).

The true target range is $R = ct_r/2$

- Consider now $t_1 > t_2$:

It follows that $t_1 + \frac{M}{prf_1} = t_2 + \frac{M+1}{prf_2}$, thus we get $M = \frac{(t_2 - t_1) + T_2}{T_1 - T_2}$ and $t_{r1} = MT_1 + t_1$

The true target range is now $R = ct_{r1}/2$

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2.5 Pulse Radar Waveform

Strategies to resolve range ambiguity

- At last, in the case $t_1 = t_2$:
The target is in the first ambiguity range, then $t_{r2} = t_1 = t_2$.

The true target range is $R = ct_{r2}/2$

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2.5 Pulse Radar Waveform

General remarks on range resolution attainable with a pulse radar

- Without intra-pulse modulation:

$$\Delta R \geq c \tau / 2$$

where τ is the pulse duration

(ex: $\tau = 1 \mu s \rightarrow 150 m$)

- With intra-pulse modulation and compression:

$$\Delta R \geq c / 2 B$$

where B is the pulse spectrum bandwidth

(ex: $100 MHz \rightarrow 1.5 m$)

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2.6 Minimum Detectable Signal

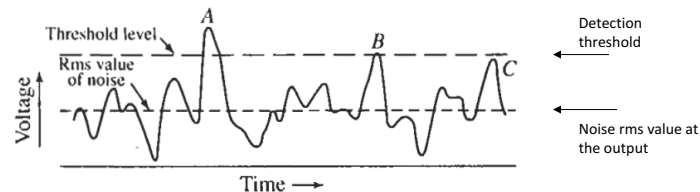
Issues to be considered:

- Detection noise: Noise statistical models;
- Probability of detection and false alarm;
- False alarm expected mean time;
- Coherent and non-coherent integration detection;
- Use of matched filters and correlators.

2.6 Minimum Detectable Signal

Noise statistic model

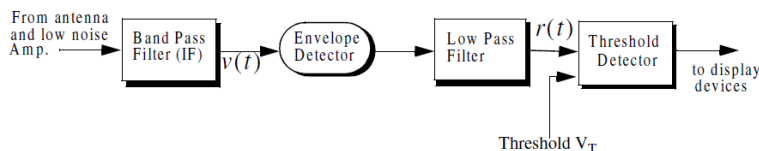
Noise envelope at the receiver output in the absence of the signal



- The noise amplitude level at the output is assumed to follow a Rayleigh distribution.
- Under the situations considered in the picture:
 - A - False detection (false alarm) if the peak value is due to noise;
 - B - Marginal detection;
 - C - No detection, if the peak value is due to a return from a target with not enough power level, or masked by noise in destructive interference.

2.6 Minimum Detectable Signal

Envelope detection with threshold decision



At the IF pass-band filter output the voltage $v(t)$ is:

$$v(t) = r(t) \cos(\omega_0 t - \phi(t)) = v_I(t) \cos \omega_0 t + v_Q(t) \sin \omega_0 t$$

Since $\cos(A - B) = \cos(A) \cos(B) + \sin(A) \sin(B)$

$$v_I(t) = r(t) \cos \phi(t) \quad (\text{In-phase component})$$

$$v_Q(t) = r(t) \sin \phi(t) \quad (\text{Quadrature component})$$

where $r(t) = \sqrt{v_I^2(t) + v_Q^2(t)}$ is the to envelope, and the phase is $\phi(t) = \tan^{-1}(v_Q/v_I)$.

2.6 Minimum Detectable Signal

Envelope detection with threshold decision

A target is detected when $r(t)$ exceeds a threshold level V_T , considering the following assumptions:

$$\begin{aligned} n(t) > V_T & \quad \text{False alarm/ false detection} \\ s(t) + n(t) > V_T & \quad \text{True detection} \end{aligned}$$

The filter output envelope is a complex random variable composed either by noise, or by noise plus signal, the latter being a sinusoidal signal with amplitude A .

- In the first case:

$$v_I(t) = n_I(t)$$

$$v_Q(t) = n_Q(t)$$

2.6 Minimum Detectable Signal

Envelope detection with threshold decision

- In the second case:

$$v_i(t) = A + n_i(t) = r(t) \cos \phi(t) \Rightarrow n_i(t) = r(t) \cos \phi(t) - A$$

$$v_q(t) = n_q(t) = r(t) \sin \phi(t)$$

where $n_i(t)$ and $n_q(t)$ are un-correlated variables with null expected value and have Gaussian distribution with variance ψ .

The joint probability distribution is then given by:

$$\begin{aligned} f(n_i, n_q) &= \frac{1}{2\pi\psi} \exp\left(-\frac{n_i^2 + n_q^2}{2\psi}\right) \\ &= \frac{1}{2\pi\psi} \exp\left(-\frac{(r \cos \phi - A)^2 + (r \sin \phi)^2}{2\psi}\right) \end{aligned}$$

2.6 Minimum Detectable Signal

Envelope detection with threshold decision

The probability density function $p(r)$, is obtained after integration on the phase ϕ :

$$p(r) = \int_0^{2\pi} f(r, \phi) d\phi = \frac{r}{\psi} \exp\left(-\frac{r^2 + A^2}{2\psi}\right) \frac{1}{2\pi} \int_0^{2\pi} \exp\left(\frac{rA \cos \phi}{\psi}\right) d\phi$$

Integration in ϕ can be made using the zeroth order modified Bessel function:

$$I_0(\beta) = \frac{1}{2\pi} \int_0^{2\pi} \exp(\beta \cos \theta) d\theta$$

Finally the Rice distribution function is obtained:

$$p(r) = \frac{r}{\psi} \exp\left(-\frac{r^2 + A^2}{2\psi}\right) I_0\left(\frac{rA}{\psi}\right)$$

2.6 Minimum Detectable Signal

Envelope detection with threshold decision

The joint probability density function in terms of $r(t)$, $\phi(t)$ is then given by

$$f(r, \phi) = f(n_i, n_q) |J|$$

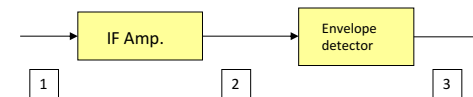
$$[J] = \begin{bmatrix} \frac{\partial n_i}{\partial r} & \frac{\partial n_i}{\partial \phi} \\ \frac{\partial n_q}{\partial r} & \frac{\partial n_q}{\partial \phi} \end{bmatrix} = \begin{bmatrix} \cos \phi & -r \sin \phi \\ \sin \phi & r \cos \phi \end{bmatrix} \quad \text{with Jacobian } |J| = r(t)$$

Substituting in the previous expressions, we get:

$$f(r, \phi) = \frac{1}{2\pi\psi} \exp\left(-\frac{r^2 + A^2}{2\psi}\right) \exp\left(\frac{rA \cos \phi}{\psi}\right)$$

2.6 Minimum Detectable Signal

Absence of Signal (noise only):



In 1: White gaussian noise;

In 2: Narrow-band gaussian noise; gaussian probability density function:

$$p(n) = \frac{1}{\sqrt{2\pi\chi}} e^{-\frac{n^2}{2\chi}}, \quad \langle n^2 \rangle = \chi$$

In 3: at the output of the (ideal) envelope detector, Rice distribution simplifies to the Rayleigh function:

$$p(r) = \frac{r}{\psi} e^{-\frac{r^2}{2\psi}} \quad \text{where} \quad \langle r^2 \rangle = 2\psi$$

2.6 Minimum Detectable Signal

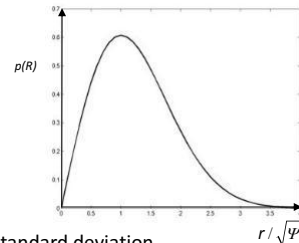
Rayleigh distribution

Probability density function $p(r) = \frac{r}{\psi} e^{-\frac{r^2}{2\psi}}$

Expected (average) value $\langle r \rangle = \int_0^\infty r p(r) dr = \sqrt{\frac{\pi}{2}\psi}$

Mean square value (power)

$$\langle r^2 \rangle = \int_0^\infty r^2 \cdot \frac{r}{\psi} e^{-\frac{r^2}{2\psi}} dr = 2\psi$$



Standard deviation

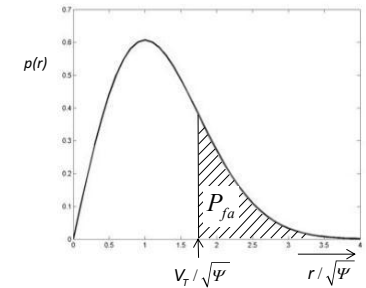
$$\sigma = \sqrt{\langle r^2 \rangle - \langle r \rangle^2} = \sqrt{2\psi - \frac{\pi}{2}\psi} = 0.655\sqrt{\psi}$$

2.6 Minimum Detectable Signal

Probability of false alarm or false detection

Assuming that a detection threshold is chosen, P_{fa} is given by:

$$P_{fa} = P(r > V_T) = \int_{V_T}^\infty p(r) dr = \int_{V_T}^\infty \frac{r}{\psi} e^{-\frac{r^2}{2\psi}} dr = e^{-\frac{V_T^2}{2\psi}}$$

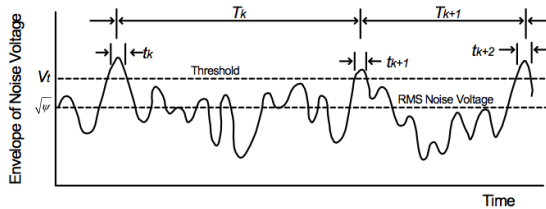


Note: $\frac{V_T^2}{2\psi} = (SNR)_T$

is the detection threshold-to-noise ratio.

2.6 Minimum Detectable Signal

False alarm time



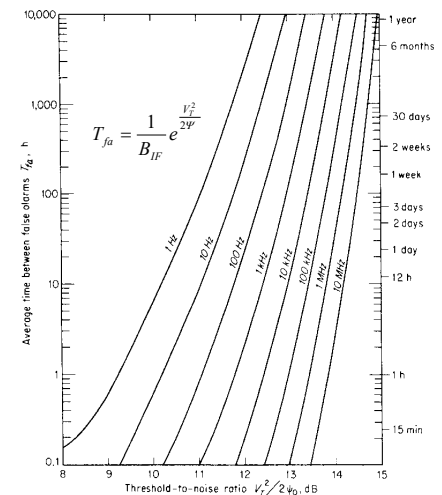
Definition: $T_{fa} = \langle T_k \rangle$ $T_{fa} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N T_k$

Assuming: $P_{fa} = \frac{\langle T_k \rangle}{T_{fa}} = \frac{\langle T_k \rangle}{T_{fa}} \approx \frac{1}{B_{IF} T_{fa}}$

Then: $T_{fa} = \frac{1}{P_{fa} \cdot B_{IF}} = \frac{1}{B_{IF}} e^{\frac{V_T^2}{2\psi}}$
 $V_T = \sqrt{2\psi \ln(1/P_{fa})}$
 Relations between false alarm time, probability of false alarm, noise power, detection bandwidth, and normalized voltage threshold:

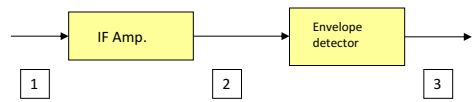
2.6 Minimum Detectable Signal

Average time between false alarms vs threshold to noise ratio



2.6 Minimum Detectable Signal

Detection (with signal)



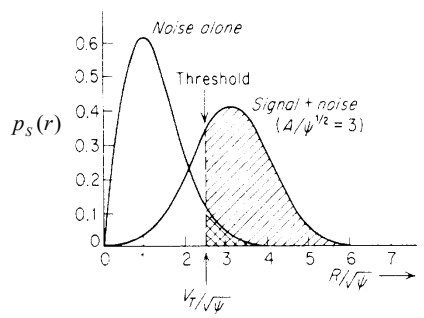
Assumptions:

- In 1: Sinusoidal signal with added white Gaussian noise;
- In 2: Signal plus narrow-band Gaussian noise;
- In 3: Envelope detector output, follows Rice distribution:

$$p_s(r) = \frac{r}{\psi} e^{-\frac{r^2+A^2}{2\psi}} I_0\left(\frac{rA}{\psi}\right)$$

2.6 Minimum Detectable Signal

Probability of detection



$$p_s(r) = \frac{r}{\psi} e^{-\frac{r^2+A^2}{2\psi}} I_0\left(\frac{rA}{\psi}\right)$$

Probability of detection, as function of the threshold, V_T

$$P_d = \int_{V_T}^{\infty} p_s(r) dr$$

2.6 Minimum Detectable Signal

Probability of detection

The probability of detection can be expressed by the integral of the probability density function between V_T and infinite:

$$P_d = \int_{V_T}^{\infty} p_s(R) dR$$

Using the probability density function, p_s , we can, alternatively, write:

$$P_d = f\left(\frac{V_T}{\psi}, \frac{A}{\psi}\right)$$

or

$$P_d = g(P_{fa}, SNR)$$

because $\frac{V_T}{\psi}$ relates with P_{fa} and $\frac{A}{\psi}$ with SNR .

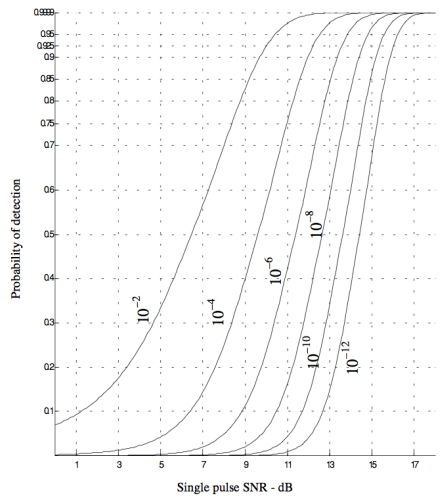
2.6 Minimum Detectable Signal

Probability of detection

$$P_d = \int_{V_T}^{\infty} p_s(r) dr = f\left(\frac{V_T}{\psi}, \frac{A}{\psi}\right) = g(P_{fa}, SNR)$$

Recall:

$$p_s(r) = \frac{r}{\psi} e^{-\frac{r^2+A^2}{2\psi}} I_0\left(\frac{rA}{\psi}\right)$$



2.6 Minimum Detectable Signal

Probability of detection

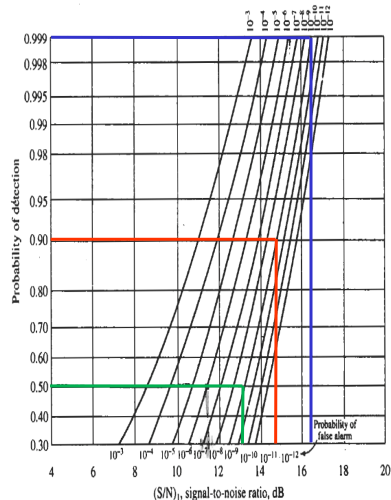
$$P_d = \int_{V_T}^{\infty} p_S(r) dr$$

$$= f\left(\frac{V_T}{\psi^{1/2}}, \frac{A}{\psi^{1/2}}\right)$$

$$= g(P_{fa}, SNR)$$

Recall:

$$p_S(r) = \frac{r}{\psi} e^{-\frac{r^2 + A^2}{2\psi}} I_0\left(\frac{rA}{\psi}\right)$$



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2.6 Minimum Detectable Signal

SNR (dB) per pulse vs P_{fa} and P_d

P_D	P_{fa}										
	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}	10^{-11}	10^{-12}	
.1	4.00	6.19	7.85	8.95	9.94	10.44	11.12	11.62	12.16	12.65	
.2	5.57	7.35	8.75	9.81	10.50	11.19	11.87	12.31	12.85	13.25	
.3	6.75	8.25	9.50	10.44	11.10	11.75	12.37	12.81	13.25	13.65	
.4	7.87	8.85	10.18	10.87	11.56	12.18	12.75	13.25	13.65	14.00	
.5	8.44	9.45	10.62	11.25	11.95	12.60	13.11	13.52	14.00	14.35	
.6	8.75	9.95	11.00	11.75	12.37	12.88	13.50	13.87	14.25	14.62	
.7	9.56	10.50	11.50	12.31	12.75	13.31	13.87	14.20	14.59	14.95	
.8	10.18	11.12	12.05	12.62	13.25	13.75	14.25	14.55	14.87	15.25	
.9	10.95	11.85	12.65	13.31	13.85	14.25	14.62	15.00	15.45	15.75	
.95	11.50	12.40	13.12	13.65	14.25	14.64	15.10	15.45	15.75	16.12	
.98	12.18	13.00	13.62	14.25	14.62	15.12	15.47	15.85	16.25	16.50	
.99	12.62	13.37	14.05	14.50	15.00	15.38	15.75	16.12	16.47	16.75	
.995	12.85	13.65	14.31	14.75	15.25	15.71	16.06	16.37	16.65	17.00	
.998	13.31	14.05	14.62	15.06	15.53	16.05	16.37	16.7	16.89	17.25	
.999	13.62	14.25	14.88	15.25	15.85	16.13	16.50	16.85	17.12	17.44	
.9995	13.84	14.50	15.06	15.55	15.99	16.35	16.70	16.98	17.35	17.55	
.9999	14.38	14.94	15.44	16.12	16.50	16.87	17.12	17.35	17.62	17.87	

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2.6 Minimum Detectable Signal

Albersheim's approximation

Albersheim proposed the following empirical expression, linking (S/N) , P_d and P_{fa} :

$$SNR = A + 0.12AB + 1.7B$$

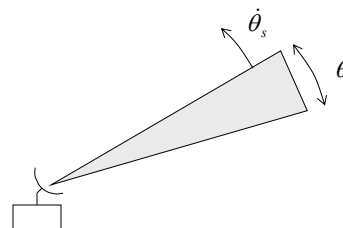
where $A = \ln(0.62/P_{fa})$ and $B = \ln(P_d/(1-P_d))$

Notes:

- SNR is in linear units, not in dB;
- The error is less than 0.2 dB for P_{fa} in the range 10^{-3} e 10^{-7} and P_d in the range 0.1 e 0.9.

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2.7 Pulse Integration



Illumination time:

$$T_{illum} = \frac{\theta_B}{\theta_s}$$

Number of returned pulses:

$$n = T_{illum} \times prf$$

$$= \frac{\theta_B prf}{\theta_s} = \frac{\theta_B prf}{6\omega_{rpm}}$$

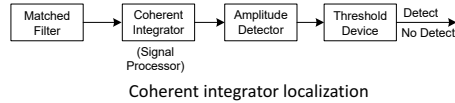
Integration can be:

- Coherent (pre-detection);
- Non-coherent (post-detection).

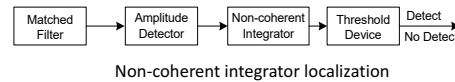
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2.7 Pulse Integration

Coherent and non-coherent integration



Coherent integrator localization



Non-coherent integrator localization

Notes:

Analog integration uses analog filters;

Digital integration usually use FFT processors.

2.7 Pulse Integration

Multiple pulse integration detection

To reduce the probability of false alarms, several pulses can be combined before a detection decision is made:

- **Coherent integration:** Pulses are added up before the envelope detector;
- **Non-coherent integration:** After each pulse envelope detection the results are added up before decision;
- **Binary integration:** Each pulse is compared to a threshold level and the number of crosses M, is used in the decision criterion;
- **Cumulative detection:** Particular case of binary integration, with $M = 1$.

2.7 Pulse Integration

Multiple pulse integration detection

Consider binary integration, for instance, in which the criterion is based on M detections on N attempts, $N > M$. The probability of having k and only k detections is

$$P_d(k, N) = \binom{N}{k} P_d^k (1 - P_d)^{N-k}$$

where P_d is the probability of a true detection;

Then, the probability of a global true detection is

$$\tilde{P}_d = \sum_{k=M}^N \binom{N}{k} P_d^k (1 - P_d)^{N-k}$$

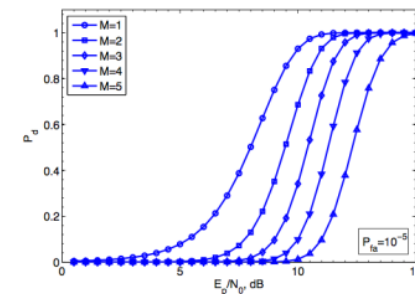
Also, the probability of a global false alarm is

$$\tilde{P}_{fa} = \sum_{k=M}^N \binom{N}{k} P_{fa}^k (1 - P_{fa})^{N-k}$$

where P_{fa} stands for only one attempt

2.7 Pulse Integration

Multiple pulse integration probability of detection



Probability of a true detection for $N=5$ and M crosses binary integration detection when the probability of false alarm per attempt is $P_{fa} = 10^{-5}$.

2.7 Pulse Integration

Integrator efficiency

When n pulses are integrated with an ideal integrator, all with the same $(SNR)_1$, the SNR value after integration is exactly n times $(SNR)_1$. Then, the required $(SNR)_{1,n}$ per pulse to produce the same probability of detection and false alarms is

$$(SNR)_{1,n} = \frac{1}{n} (SNR)_1$$

When the same n pulses are integrated with post detection, the resulting SNR is lower, due to part of the signal energy be converted to noise in the envelope detector (a non-linear device).

2.7 Pulse Integration

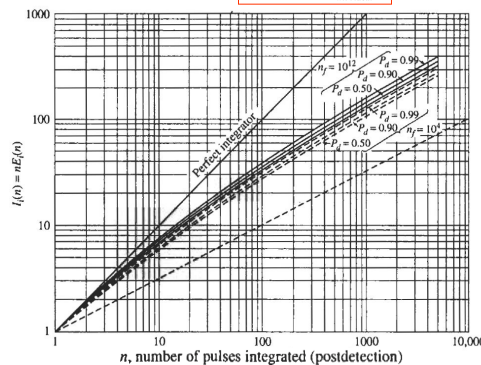
Integrator efficiency

Efficiency of a non-coherent integrator

$$E_i(n) = \frac{(SNR)_1}{n(SNR)_{n,1}}$$

$$E_i(n) = \frac{(SNR)_{with\ int}}{(SNR)_{with\ ideal\ int}}$$

$$\frac{1}{\sqrt{n}} \leq E_i(n) \leq 1$$



Improvement factor
Integration loss

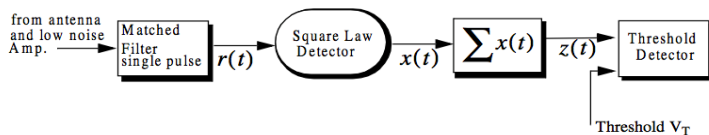
$$I_i(n) = nE_i(n)$$

$$L_i(n) = 10 \log \left[\frac{1}{E_i(n)} \right]$$

Improvement factor of a quadratic integrator as a function of the number of integrated pulses and the probability of detection. The false alarm number n_f is defined as the inverse of P_{fa}

2.7 Pulse Integration

Non-coherent integrator



A non-coherent integrator is usually implemented as an envelope detector known as **quadratic integrator**.

The output after the n^{th} pulse is proportional to the input square.

2.7 Pulse Integration

Empirical expressions for a non-coherent integrator

Non-coherent integrator

Empirical estimate of the improvement factor as a function of the number of integrated pulses, and P_{fa} and P_d :

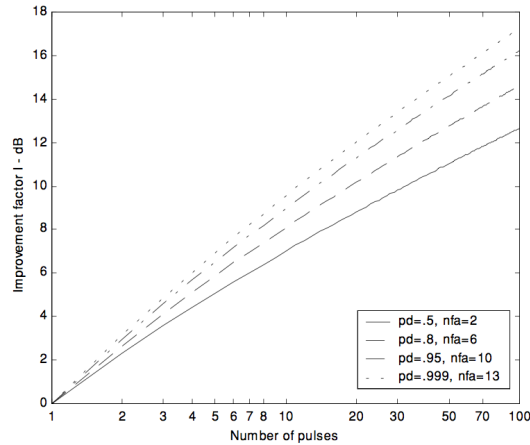
$$I(n)_{dB} = 6.79(1 + 0.235P_d) \left(1 + \frac{\log_{10}(1/P_{fa})}{46.6} \right) \log_{10}(n)$$

$$\times (1 - 0.14 \log_{10}(n) + 0.01831(\log_{10}(n))^2)$$

(error less than 0.8 dB).

2.7 Pulse Integration

Empirical expressions for a non-coherent integrator



Improvement factor as a function of the number of integrated pulses, the probability of detection P_d and the number of false alarm $nfa = 1/P_{fa}$, sometimes, as in the picture, it is used $nfa = \log_{10}(1/P_{fa})$

2.7 Pulse Integration

Radar range equation with and without integrator

Range without integration

$$R_{\max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 K T_{eq} \Delta f (SNR)_1} \right]^{1/4} \quad (\Delta f = B_n)$$

Range with integration

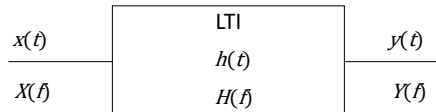
$$R_{\max} = \left[\frac{P_t G A_e \sigma n E_i(n)}{(4\pi)^2 K T_{eq} \Delta f (SNR)_{1,n}} \right]^{1/4}$$

Note: $(SNR)_1$ depends on the probability of detection and false alarm; $(SNR)_{1,n}$ also depends on the integration type and on the number of integrated pulses.

2.8 Matched filter detection

Detection using Linear Filters

Linear time invariant (LTI) systems theory



$$Y(f) = X(f)H(f) \quad H(f) = \mathcal{F}[h(t)] \quad h(t) = \mathcal{F}^{-1}[H(f)]$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^t x(\tau)h(t - \tau) d\tau$$

At an instant in time $t = t_1$

$$y(t_1) = \int_{-\infty}^{\infty} X(f)H(f)e^{j2\pi f t_1} df$$

2.8 Matched filter detection

Maximum signal amplitude (peak value) at a filter output

Schwartz inequality:

$$\left| \int P^* Q dx \right|^2 \leq \int P^* P dx \int Q^* Q dx$$

equal sign if, and only if, $P = \text{constant} \times Q$

with $P^* = S_i(f)e^{j2\pi f t_1} \quad Q = H(f)$

At the input: $s_i(t) \equiv x(t) \quad \mathcal{F}[x(t)] \equiv S_i(f)$

At the output: $s_o(t) \equiv y(t) \quad \mathcal{F}[y(t)] \equiv S_o(f) \equiv S_i(f)H(f)$

$$s_o^2(t_1) = \left| \int S_i(f)H(f)e^{j2\pi f t_1} df \right|^2 \leq \int |S_i(f)|^2 df \int |H(f)|^2 df$$

2.8 Matched filter detection

Peak SNR at the filter output

Output noise power

$$\frac{\langle n_0^2 \rangle}{R} = \frac{KT}{2} \int |H(f)|^2 df \quad (\text{matched load } R, \text{ is assumed})$$

Energy at the output

$$E = \frac{1}{R} \int s_0^2(t) dt = \frac{1}{R} \int |S_i(f)|^2 df$$

Schwartz inequality implies for SNR

$$\frac{s_0^2(t_i)}{\langle n_0^2 \rangle} \leq \frac{2E}{KT}$$

Note: This value of SNR takes as reference the instant peak power of the signal and the expected value of noise power.

2.8 Matched filter detection

Matched filter and pulse response

$$\text{If } H(f) = G S_i^*(f) e^{-j2\pi f t_i}$$

the filter is referred to as matched filter.

Then the peak output value of SNR is

$$\frac{2E}{KT}$$

The filter pulse response is

$$h_a(t) = G \int S_i^*(f) e^{-j2\pi f t_i} e^{j2\pi f t} df$$

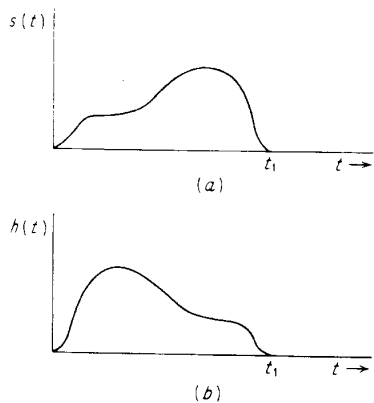
$\underbrace{\hspace{10em}}_{H_a(f)}$

For real signals $S_i^*(f) = S_i(-f)$ we get

$$h_a(t) = G S_i(t_1 - t)$$

2.8 Matched filter detection

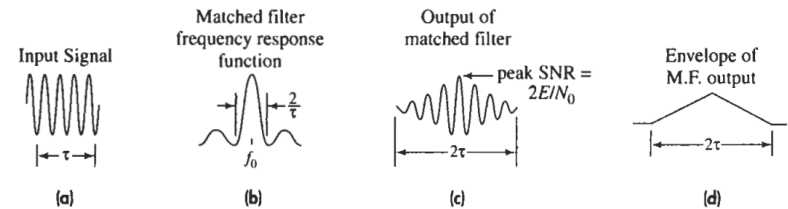
Matched filter and pulse response



- The input response, and the signal to be detected, exhibit inverted time variation;
- To be possible to realize a matched filter, the signal to be detected must have finite duration and energy;
- The maximum amplitude, at the output, occurs after the input signal ends;
- A time delay is necessary to allow realizability; Else the pulse response would start before the signal appearance at the input.

2.8 Matched filter detection

Matched filters for the detection of rectangular shaped pulses



- a) Rectangular pulse (RF) ; b) frequency response of the matched filter;
c) Time response to the pulse; d) envelope of the matched filter output

2.8 Matched filter detection

Efficiency of non-matched filters

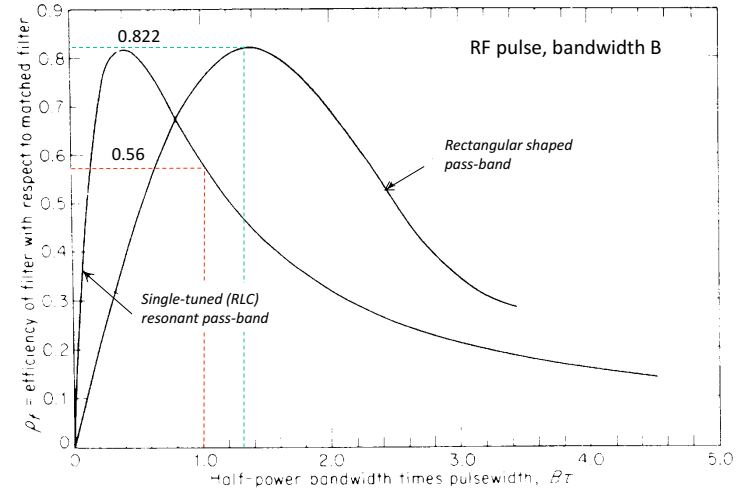
Input signal	Filter	Optimal $B_{RF}\tau$	(SNR) _(dB) loss when compared to a matched filter
Rectangular pulse	rectangular	1.37	0.85
Rectangular pulse	Gaussian	0.72	0.49
Gaussian pulse	rectangular	0.72	0.49
Gaussian pulse	Gaussian	0.44	0 (matched)
Rectangular pulse	RLC pass-band resonance circuit	0.4	0.88
Rectangular pulse	2 cascade RLC tuned circuit	0.613	0.56
Rectangular pulse	5 cascade RLC tunes filters	0.672	0.5

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2.8 Matched filter detection

RLC and rectangular filters comparison



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2.9 Correlator detection

Cross-correlation

At the input: signal plus noise $x(t) = s_i(t) + n_i(t)$

Matched filter output

$$\begin{aligned}
 y(t) &= \int x(\tau)h(t-\tau)d\tau \\
 &= G \int x(\tau)s_i(t_1-t+\tau)d\tau \\
 &= G R_{x,s_i}(t_1-t)
 \end{aligned}$$

Note: cross-correlation between real functions $v(t)$ and $w(t)$

$$R_{v,w}(t) = \int v(\tau)w(\tau+t)d\tau$$

If $w(t)=v(t)$, $R_v(t)=R_{vv}(t)$ is the self-correlation of $v(t)$

At the filter output it is obtained the cross-correlation between the signal corrupted by noise and a delayed replica of the signal inverted in time (pulse response of the matched filter)

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2.9 Correlator detection

Cross-correlation

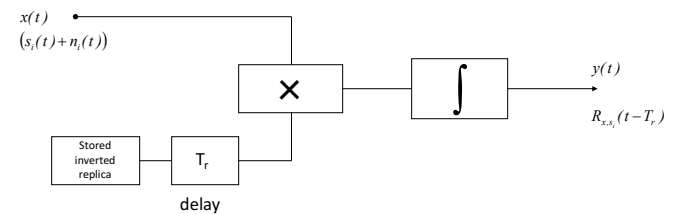
$$y(t) = G R_{s_i}(t-t_1) + G R_{n,s_i}(t-t_1)$$

Note: $R_v(-t) = R_v(t)$ real signals

Signal to be detected cross-correlation

Filtered noise

Detection can be performed, in an equivalent way, using a correlator



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2.9 Correlator detection

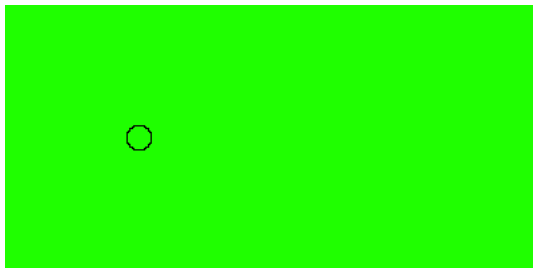
Matched filter vs correlator

Notes:

- The correlator tests the presence of the target at receiving instants T_r . Thus, the operation is repeated successively, which corresponds to a time discretization. The searching time can be reduced using parallel processing. The use of correlators can be highly efficient making use of appropriate digital processing algorithms.
- Correlation detection can be seen as a time filtering technique whereas matching filter detection is a frequency domain filtering.

2.9 Radar Cross Section

Radiation and scattering



Total field

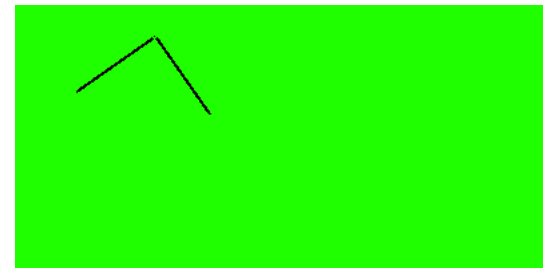
Re-radiated or scattered field

2.9 Radar Cross Section

- Radiation and scattering;
- Target radar cross section;
- Swerling fluctuation models;
- Probability of detection with RCS fluctuations;
- Range equation considering fluctuations.

2.9 Radar Cross Section

Radiation and scattering



Total field

Re-radiated or scattered field

2.9 Radar Cross Section

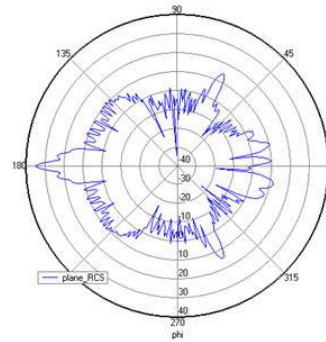
Radiation and scattering

Incident field E_{inc}



Total field $E_T = E_{inc} + E_s$

scattered
or re-radiated field



2.9 Radar Cross Section

Total RCS

- The previous definition considers the RCS mono-static case, or back-scatter RCS, or simply the RCS. In the definition, a direction is assumed, and also one reference polarization.
- Assuming a spherical coordinate system, the RCS is a function of the aspect angle. If the total reflected power in all directions away from the target (θ_s, ϕ_s) is to be considered, we obtain the total target cross-section for a particular incident direction:

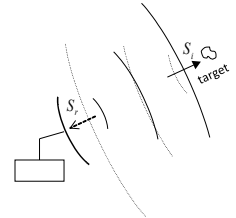
$$\sigma_t = \frac{1}{4\pi} \int_{\phi_s=0}^{2\pi} \int_{\theta_s=0}^{\pi} \sigma(\theta_s, \phi_s) \sin(\theta_s) d\theta_s d\phi_s$$

2.9 Radar Cross Section

Equivalent radar cross section

Definition:

$$\sigma = \frac{\text{re-radiated power per solid angle unit}}{\text{incident power density at the target}}$$



The mono-static RCS definition considers the re-radiated power per solid unit angle in the backscatter direction

$$P_{rad} = \sigma S_i = \frac{\sigma E_i^2}{2Z}$$

$$S_r = \frac{P_{rad}}{4\pi R^2} = \frac{E_r^2}{2Z}$$

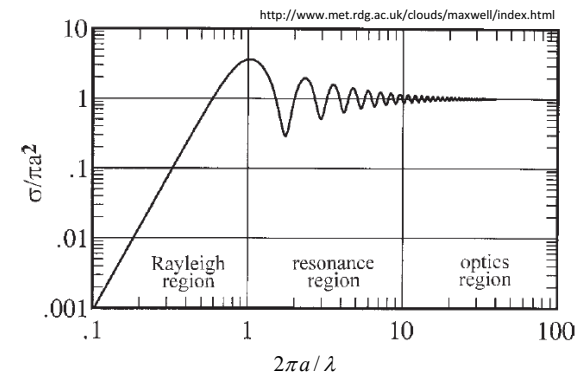
$$\therefore \sigma = 4\pi R^2 \left| \frac{E_r}{E_i} \right|^2$$

Note: σ depends on:

- material
- form and dimensions
- aspect angles
- frequency
- polarization

2.9 Radar Cross Section

Example: Sphere



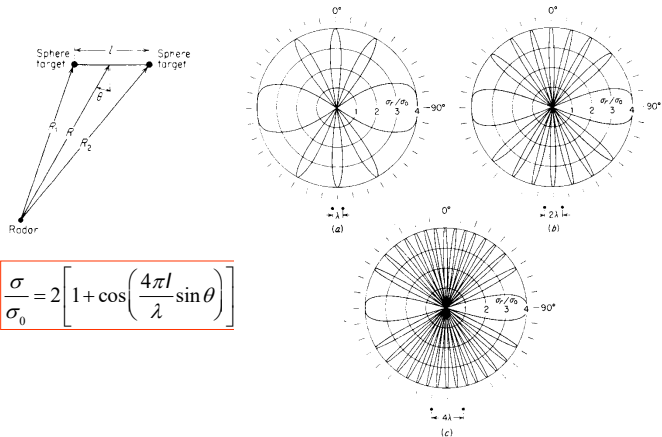
Note: Rayleigh limit for a metallic sphere

$$\sigma_{Rayleigh} = 9\pi a^2 \left(\frac{2\pi a}{\lambda} \right)^4$$

Sphere cross-section as a function of the perimeter of the maximum circle, to wavelength

2.9 Radar Cross Section

Example: Two-point target



$$\frac{\sigma}{\sigma_0} = 2 \left[1 + \cos \left(\frac{4\pi l}{\lambda} \sin \theta \right) \right]$$

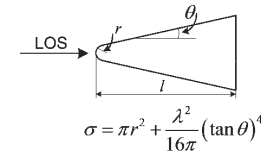
- a) $l = \lambda$
- b) $l = 2\lambda$
- c) $l = 4\lambda$

2.9 Radar Cross Section

Example: Conical st



Figure 2-6 – Cone Geometry



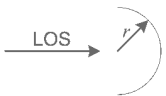
$$\sigma = \pi r^2 + \frac{\lambda^2}{16\pi} (\tan \theta)^4$$

Figure 2-7 – Ideal Reentry Vehicle RCS – Nose-on Aspect

2.9 Radar Cross Section

Example: Reflectors with simple shapes

Circular Reflector



$$\sigma = \frac{4\pi}{\lambda^2} (\pi r^2)^2$$

Triangular Corner Reflector



$$\sigma = \frac{4\pi r^4}{3\lambda^2}$$

Chaff Dipole

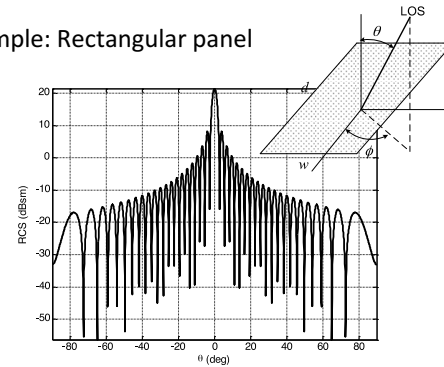


$$\sigma = 0.93\lambda^2$$

RCSs of Some Simple Shapes

2.9 Radar Cross Section

Example: Rectangular panel



RCS of a Flat Plate

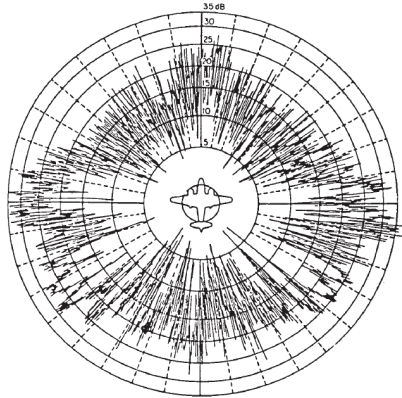
$$\sigma = \frac{(kdw)^2}{\pi} \text{sinc}^2 [kd \sin \theta \cos \phi] \text{sinc}^2 [kw \sin \theta \sin \phi] \times 0.5 \left[\sqrt{1 - \sin^2 \theta \cos^2 \phi} + \sqrt{1 - \sin^2 \theta \sin^2 \phi} \right]$$

where

$$\text{sinc}(x) = \frac{\sin(\pi x)}{(\pi x)}$$

2.9 Radar Cross Section

Example: Aircraft



Experimental RCS of a B-26 measured at 3 GHz

2.9 Radar Cross Section

Typical radar cross sections at microwaves

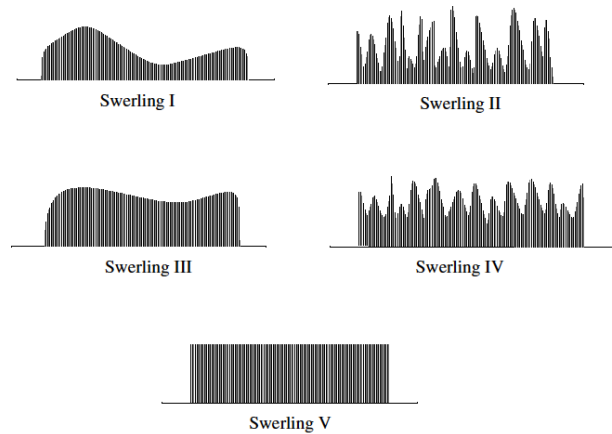
Target	RCS [m ²]
Conventional missile	0.5
Small size airplane	1
Small size fighter	2
Big size fighter	6
Medium size bomber or jet airliner	20
Big size bomber or large airliner	40
Jumbo jet	100

2.11 RCS Fluctuations

- The amplitude of the returns is rarely constant in practice;
- These variations can be due to atmospheric conditions, to the side-lobe antenna pattern, to equipment instabilities (receiver) or to random fluctuations of the target radar cross-section;
- To take into account the target RCS fluctuations in the range prediction, one can select a value that is not exceeded in a high percentage of the total observations (ex: 95%, 99%), characterizing, thus, a minimum RCS;
- In the case of air targets, such as airplanes helicopters and missiles, the minimum RCS is for frontal observations.

2.11 RCS Fluctuations

Swerling Models



2.11 RCS Fluctuations

Swerling models

- SW1/SW3 – Constant amplitude is assumed during a sweep and un-correlated amplitudes in consecutive sweeps (scan-to-scan fluctuations);
- SW2/SW4 – Fast fluctuations, un-correlated pulse to pulse;
- SW1/SW2 – The probability density function of a target having an equivalent reflecting area σ , is given by:

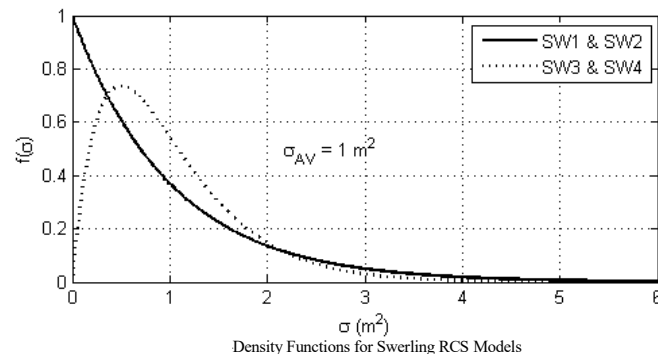
$$p(\sigma) = \frac{1}{\sigma_{av}} e^{-\frac{\sigma}{\sigma_{av}}} \quad (\sigma > 0)$$

- SW3/SW4 – Is assumed that:

$$p(\sigma) = \frac{4\sigma}{\sigma_{av}^2} e^{-\frac{2\sigma}{\sigma_{av}}} \quad (\sigma > 0)$$

2.11 RCS Fluctuations

Swerling models



2.11 RCS Fluctuations

Swerling models

- **SW1** and **SW2** models are more adequate for complex targets made of many reflectors with approximately equal individual RCS;
- **SW3** and **SW4** models are more adequate for targets that have a bigger reflector and a fair number of smaller reflectors;
- **Rice Distribution** this mathematical model was developed to the case when there is a big reflector and a small number of equal individual RCS have smaller size:

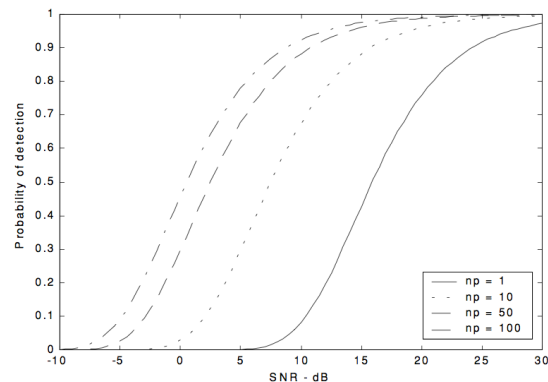
$$p(\sigma) = \frac{1+s}{\sigma_{av}} e^{-s-\frac{\sigma}{\sigma_{av}}(1+s)} I_0\left(2\sqrt{\frac{\sigma s(1+s)}{\sigma_{av}}}\right) \quad (\sigma > 0)$$

where

$$s = \frac{\text{RCS of the major reflector}}{\text{sum of the small reflectors RCS}}$$

2.11 RCS Fluctuations

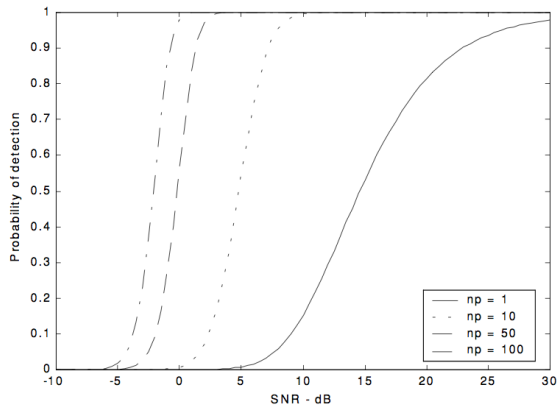
SW1 probability of detection



For the cases represented in the figure, $P_{fa}=10^{-12}$

2.11 RCS Fluctuations

SW2 probability of detection



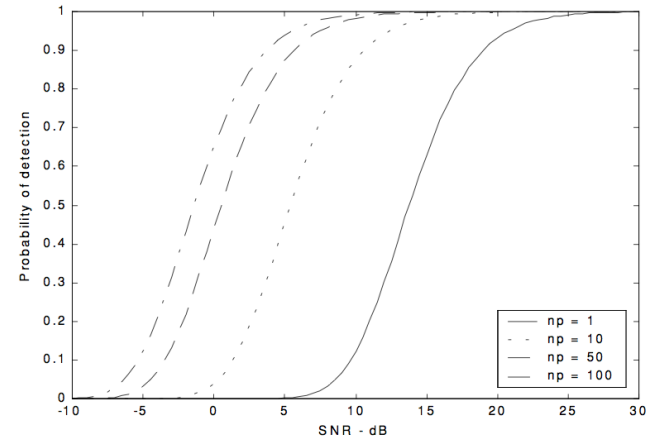
For the cases represented in the figure, $P_{fa}=10^{-9}$

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2.11 RCS Fluctuations

SW3 probability of detection



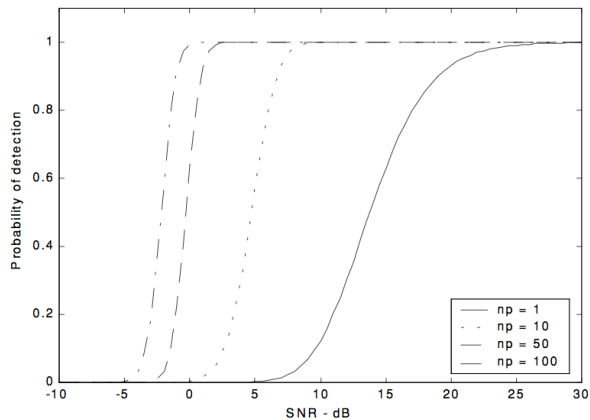
For the cases represented in the figure, $P_{fa}=10^{-9}$

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2.11 RCS Fluctuations

Probability of detection, SW4 case



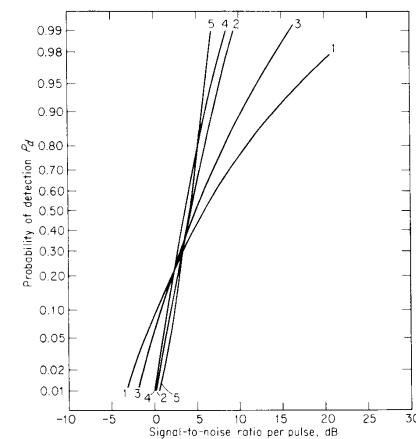
For the cases represented in the figure, $P_{fa}=10^{-9}$

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2.11 RCS Fluctuations

Comparison of probability of detection for target with Swerling models



- 1- SW1
- 2- SW2
- 3- SW3
- 4- SW4
- 5- No fluctuations, sometimes called SW5

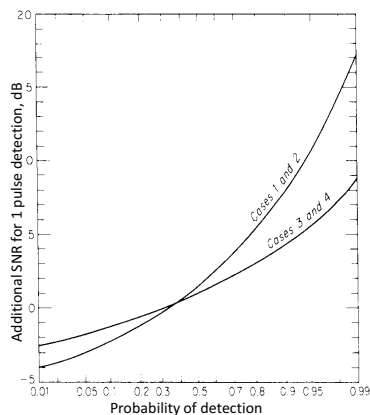
Note: P_d values shown in the figure correspond to 10 pulse integration with $P_{fa}=10^{-8}$

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2.11 RCS Fluctuations

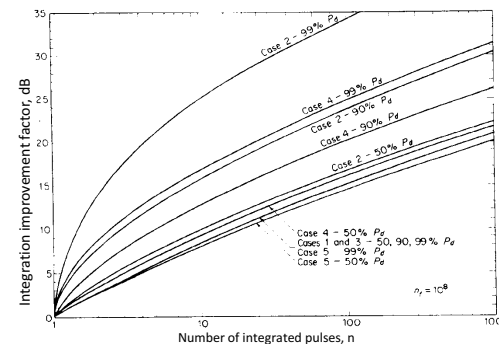
Additional SNR per pulse as a function of the probability of detection



The figure shows an estimate of the additional SNR per pulse, relative to the non-fluctuating case, required as a function of the probability of detection

2.11 RCS Fluctuations

Integration improvement factor accounting for the fluctuation model



This figure shows estimates for the integration improvement factor for SW cases as a function of the number of pulses (in the plot, $nfa=8$, $P_d=50,90$ and 99%)

2.11 RCS Fluctuations

A simplified procedure for the radar range estimation

- 1- Determine $(SNR)_1$, without integration nor fluctuations, for given P_d and P_{fa}
- 2- Make use of the correct expressions for the relevant Swerling case considering the number of integrated pulses.

Alternatively one can estimate an addition value $(SNR)_1^+$ (depending on the SW model)

$$(SNR)_1^+ (dB) = (SNR)_1 (dB) + L_{fl} (dB)$$

Then, estimate the integration improvement factor, considering the proper fluctuation model

Use the radar range equation

$$R_{max} = \left[\frac{P_t \lambda^2 G^2 \sigma_{av} n E_i(n)}{(4\pi)^3 K T_{eq} \Delta f (SNR)_1^+} \right]^{1/4}$$

$$= \left[\frac{P_t \lambda^2 G^2 \sigma_{av} n E_i(n)}{(4\pi)^3 K T_{eq} \Delta f (SNR)_1 L_{fl}} \right]^{1/4} \quad (\text{linear units})$$

2.11 RCS Fluctuations

Range in terms of the average power

Transmitted power

$$P_t, \text{ peak value; } P_{av}, \text{ average value}$$

for rectangular pulses

$$P_{av} = \frac{P_t \tau}{T_p} = P_t \tau PRF \quad \tau / T_p = \text{duty cycle}$$

Range equation

$$R_{max} = \left[\frac{P_{av} \lambda^2 G^2 \sigma_{av} n E_i(n)}{(4\pi)^3 K T_{eq} (\Delta f \tau) PRF (SNR)_1 L_{fl}} \right]^{1/4}$$

$$\approx 1$$

Note that $\Delta f \tau \approx 1$

2.12 Propagation Effects

Propagation effects to be considered

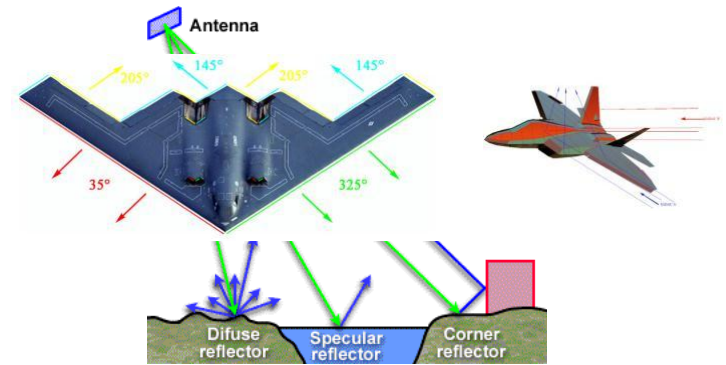
- Reflection
- Refraction
- Diffraction
- Attenuation
- Losses

Atmospheric propagation effects

- Refraction
- Anomalous propagation conditions
- Wave attenuation in the atmosphere

2.12 Propagation Effects

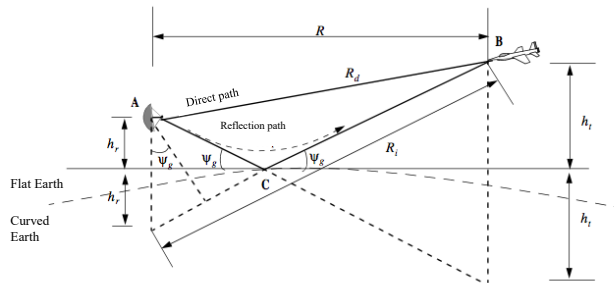
Reflection



Some different types of reflection: diffuse, specular, corner

2.12 Propagation Effects

Reflection at a plane perfect electric conductive Earth



Reflection coefficients
"smooth"

Vertical polarization

$$\Gamma_v = \frac{e \sin \psi_g - \sqrt{e - (\cos \psi_g)^2}}{e \sin \psi_g + \sqrt{e - (\cos \psi_g)^2}}$$

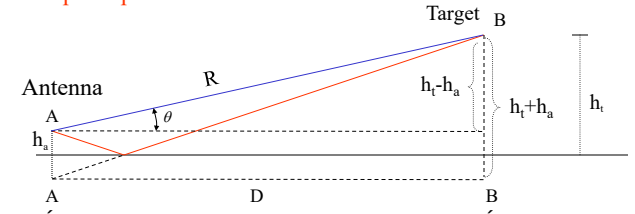
Horizontal polarization

$$\Gamma_h = \frac{\sin \psi_g - \sqrt{e - (\cos \psi_g)^2}}{\sin \psi_g + \sqrt{e - (\cos \psi_g)^2}}$$

Note: usually there is no need of considering corrections due to the Earth curvature

2.12 Propagation Effects

Reflection at a plane perfect electric conductive Earth



$\Delta r = A'B - AB$ Path difference between "direct" and "reflected ray"

$$D^2 = \overline{AB}^2 - (h_t - h_a)^2$$

$$(h_t + h_a)^2 - (h_t - h_a)^2 = \overline{A'B}^2 - \overline{AB}^2$$

$$D^2 = \overline{A'B}^2 - (h_t + h_a)^2$$

$$4h_t h_a = \Delta r \times (\overline{AB} + \overline{A'B})$$

$$\text{Phase difference } \psi = \frac{4\pi h_t h_a}{\lambda R} + \pi$$

$$\theta \approx \sin \theta \approx \frac{h_t}{R} \therefore \psi \approx \frac{4\pi h_t \theta}{\lambda} + \pi$$

2.12 Propagation Effects

Reflection at a plane perfect electric conductive Earth

At the target

$$\frac{E_T}{E_{dir}} = 1 - e^{-j\frac{4\pi h_a h_t}{\lambda R}} = 2je^{-j\frac{2\pi h_a h_t}{\lambda R}} \sin\left(\frac{2\pi h_a h_t}{\lambda R}\right) \Rightarrow F = \left| 2\sin\left(\frac{2\pi h_a h_t}{\lambda R}\right) \right|$$

$$\frac{P_{alvo}}{P_{dir}} = 4\sin^2\left(\frac{2\pi h_a h_t}{\lambda R}\right)$$

The so-called propagation factor, F, is introduced here

At the receiver

$$\frac{P_{rec}}{P_{dir}} = 16\sin^4\left(\frac{2\pi h_a h_t}{\lambda R}\right)$$

$$P_{rec} = \frac{P_i G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \times 16\sin^4\left(\frac{2\pi h_a h_t}{\lambda R}\right) = \frac{P_i G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \times F^4$$

2.12 Propagation Effects

Reflection at a plane perfect electric conductive Earth

F has a lobed structure with **maxima**

$$\frac{2\pi h_a h_t}{\lambda R} = (2n+1)\frac{\pi}{2} \quad \text{first maximum... } \theta \approx \frac{\lambda}{4h_a}$$

and **minima** for

$$\frac{2\pi h_a h_t}{\lambda R} = n\pi$$

In the case of grazing incidence

$$\text{hip: } \frac{h_t h_a}{\lambda R} \ll 1 \sin^4\left(\frac{2\pi h_a h_t}{\lambda R}\right) \approx \left(\frac{2\pi h_a h_t}{\lambda R}\right)^4$$

$$P_{rec} = \frac{4\pi P_i G^2 \sigma (h_a h_t)^4}{\lambda^2 R^8}$$

Note the dependence of the received power with R^{-8} for this case

2.12 Propagation Effects

Refraction

At normal propagation conditions

– A good approximation is considering a linear path in a deformed Earth with an equivalent radius

– where

$$a_e = ka_T \quad (a_T = 6370 \text{ km})$$

$$k = \frac{1}{1 + a_T \frac{\partial n}{\partial h}}$$

“standard” value $\frac{\partial n}{\partial h} = -4 \times 10^{-8} m^{-1}$

$$k = \frac{4}{3}$$

2.12 Propagation Effects

Refraction

Refractivity

$$N = (n-1) \times 10^6$$

$$\approx \frac{77.6}{T} \left[p + \frac{4810 e}{T} \right]$$

- p = barometric pressure, mbar 1 mm Hg = 1.332 mbar
- e = water vapor partial pressure, mbar
- T = absolute temperature

Exponential model

$$N = N_s e^{-h/H_s}$$

N_s = refractivity at the Earth surface

H_s , scale factor (typical value, H_s 7.35 km)

h, heigth (km)

2.12 Propagation Effects

Refraction

- Notes
- Distance to a target should be corrected considering the refraction index not equal to 1 (vacuum) and the non-homogeneity of the troposphere
- To deal with this issue ray-tracing should be used
- The following approximate expression can be used for targets at heights in the 15 to 65 thousand feet

$$\Delta R_m \approx 0.42 + 0.0577 R_{km} (N_s / h_{kft})^{1/2}$$

$$\Delta R_m \approx 0.42 + 0.1045 R_{km} (N_s / h_{km})^{1/2}$$

R	50	km
N	7.5	km
h	3	km
	8.7	m

ΔR_m correction (meter)

R_{km} distance to target (km) estimated time delay assuming vacuum

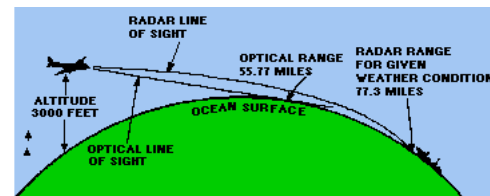
N_s refractivity at the Earth surface level

h_{kft} target height (thousand feet; 1 ft = 0.3048 m)

h_{km} target height (km)

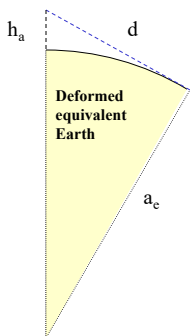
2.12 Propagation Effects

Radar horizon due to refraction



2.12 Propagation Effects

Radar horizon due to refraction



Radar horizon is generally higher than the geometric, due to refraction

$$(a_e + h_a)^2 = d^2 + a_e^2$$

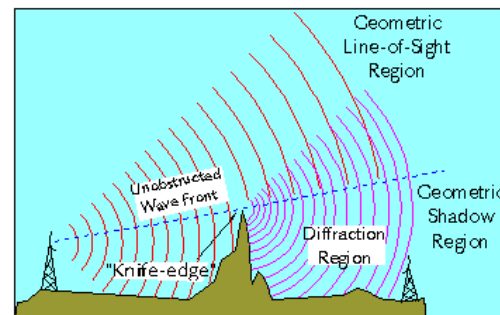
$$h_a^2 + 2a_e h_a = d^2$$

$$d \approx \sqrt{2a_e h_a}$$

$$= \sqrt{2ka_r h_a}$$

2.12 Propagation Effects

Diffraction

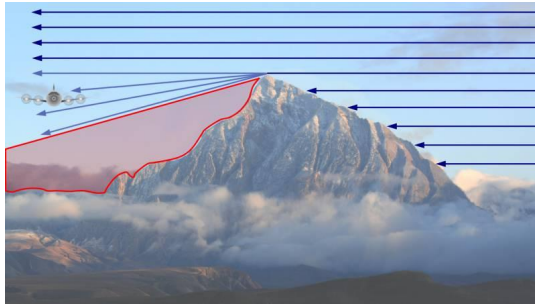


knife-edge effect

Diffraction occurs when an EM wave has an obstacle in its path. The phenomenon is enhanced when the wavelength compares to the object dimensions

2.12 Propagation Effects

Diffraction

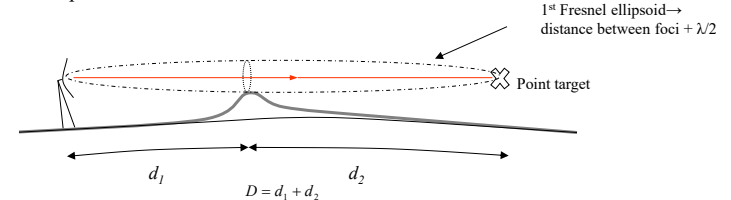


Diffraction occurs when an EM wave has an obstacle in its path. The phenomenon is enhanced when the wavelength compares to the object dimensions

2.12 Propagation Effects

Fresnel ellipsoid

1st Fresnel ellipsoid radius at the obstacle area



$$R_1 = 17.3 \sqrt{\frac{d_1 d_2}{fD}}$$

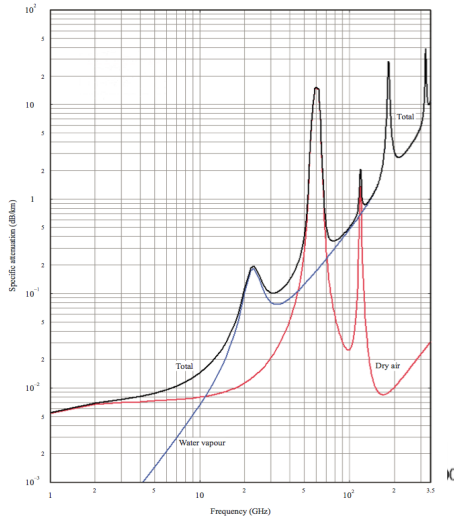
$d_{1,2}, D$ (km)
 R_1 (m)
 f (GHz)

Ex: $d_1=20$ km, $d_2=30$ km, $f=10$ GHz $\rightarrow R_1 \approx 20$ m

Note: in radio links is enough to guarantee a clear zone of $\sim 0.5 R_1$

2.12 Propagation Effects

Atmospheric attenuation



Atmospheric attenuation

In the picture
 - atm. pressure: 76 cm Hg
 - Temperature 15°C
 - 1% molecules of water vapor (7.5 g water/m³);

Notice the absorption peaks due to water and oxygen

2.12 Propagation Effects

Rain Attenuation

Specific absorption rate, γ_R (dB/km) is obtained by

$$\gamma_R = k R^\alpha$$

where R (mm/h) is the precipitation rate, and k and α are given by

$$\log k = \sum_{j=1}^3 \left[a_j \exp \left[- \left(\frac{\log f - b_j}{c_j} \right)^2 \right] \right] + m_k \log f + c_k$$

$$\alpha = \sum_{i=1}^4 \left[a_i \exp \left[- \left(\frac{\log f - b_i}{c_i} \right)^2 \right] \right] + m_\alpha \log f + c_\alpha$$

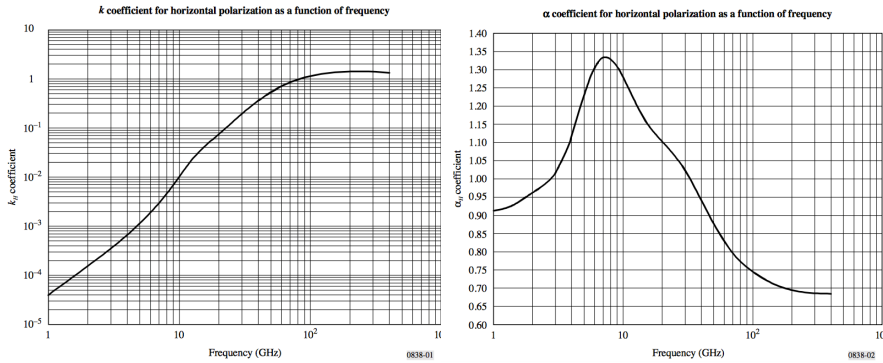
where

$k = k_H$ or k_V , and $\alpha = \alpha_H$ or α_V according to polarization

and the coefficients $a, b, c, m_b, c_k, m_\alpha$ and c_α are the next slide table

2.12 Propagation Effects

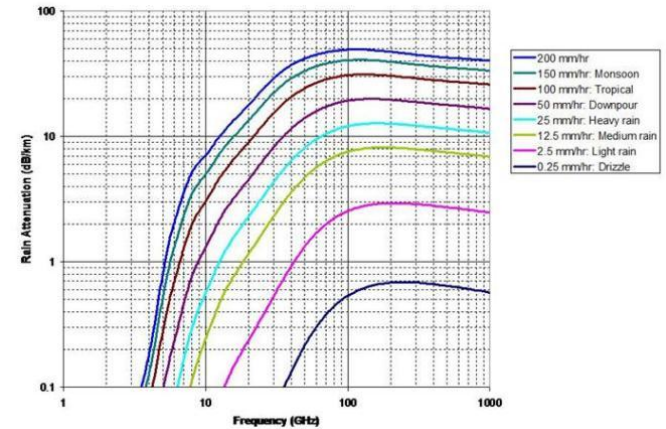
Rain Attenuation



http://www.itu.int/dms_pubrec/itu-r/rec/p/R-REC-P.838-2-200304-S!!PDF-E.pdf

2.12 Propagation Effects

Rain Attenuation



2.12 Propagation Effects

System loss

The available transmitter output power differs from the power delivered to the antenna due to losses in the waveguides, insertion loss in the duplexer, circulators, direction couplers and rotary joints as well as in other μ – wave components.

For the same reason the available power at the antenna terminals is not the same as the power delivered to the receiver

These are called system losses. A loss factor can thus be introduced L_{sys}

- Example of typical contribution to the system losses for an S-band radar, commonly used in airports:
 - Waveguides (~30 m two-way) ~1 dB
 - Duplexer ~2 dB
 - Rotary joint ~1 dB
 - Connectors and bends ~0.3 dB
 - Other devices ~0.4 dB

2.12 Propagation Effects

Range equation accounting for propagation effects and system losses

$$R_{\max} = \left[\frac{P_t \lambda^2 G^2 \sigma_{av} n E_i(n) F^4}{(4\pi)^3 K T_{eq} \Delta f (SNR)_1 L_{fl} L_p L_{sis}} \right]^{1/4}$$

Note that accounting for propagation effects and system losses to estimate the maximum can be difficult if losses are not duly characterized. In the expression above absorption due to atmospheric gases and rain are accounted in terms

Besides the considered losses we could also include losses due to: beam-shape, scanning (in the case of rapid scanning), radome (often used), and also signal processing losses