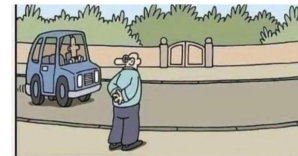


Chapter 3: CW Radar

Applications:

- Measurement of the relative speed (unmodulated CW radar): Land vehicles, projectiles, missiles... and also for sports;
- Vertical takeoff speed measurement;
- Altimeter (Frequency modulated CW);
- Dockage operation in space;
- Collision avoidance;
- Anti-intrusion alarms;
- Track illuminator for missile guiding systems (associated to a pulse Doppler radar for better performance).

3.1 Doppler effect



Apparent frequency

$$f' = f_0 \left(\frac{c \pm v_{obsv}}{c \mp v_{source}} \right)$$

Moving source

$$f' \approx f_0 \left(1 \pm \frac{v_{source}}{c} \right)$$

$$f_d = f' - f_0$$

$$= \pm f_0 \frac{v}{c}$$

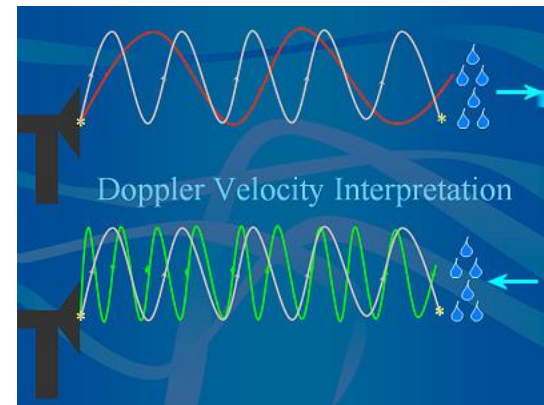
$$= \pm \frac{v}{\lambda}$$

3.1 Doppler effect



An F/A-18 Hornet passing through the sound barrier: A combination of conditions (water vapor, density and temperature) must occur to create this event.

3.1 Doppler effect



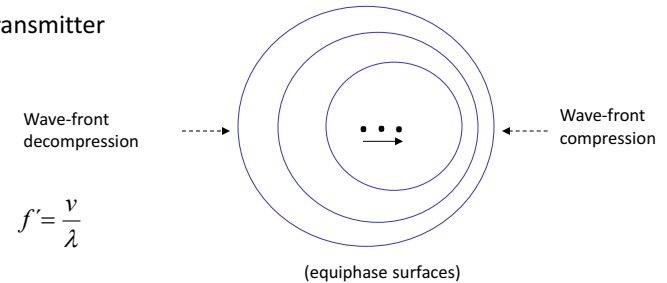
3.1 Doppler effect

Speed measurement

- Velocity information from a target can be easily obtained with a continuous wave (CW) radar, by comparing the frequency of the returned to the transmitted signal.
- The difference between these frequencies, or Doppler shift frequency, is directly proportional to the radial approaching or moving away velocity of a particular target relative to the transmitter.
- Often, the Doppler shift value is of the order or less than tens of kHz, in many cases in the audio band.

3.1 Doppler effect

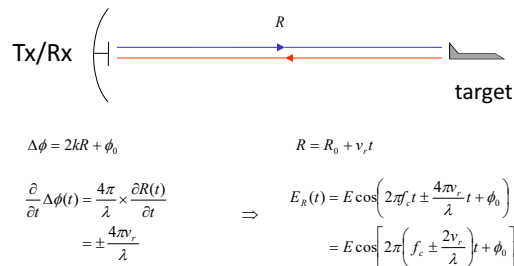
Moving transmitter



- In the case of a mono-static radar the Doppler effect is double because it occurs both in the go and return path

$$f_d = \frac{2v}{\lambda}$$

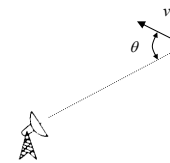
3.1 Doppler effect



$f_d = \frac{2v_r}{\lambda}$	Doppler frequency shift
f_c	Transmitted frequency
$f_c \pm f_d$	Received frequency

3.1 Doppler effect

Radial velocity



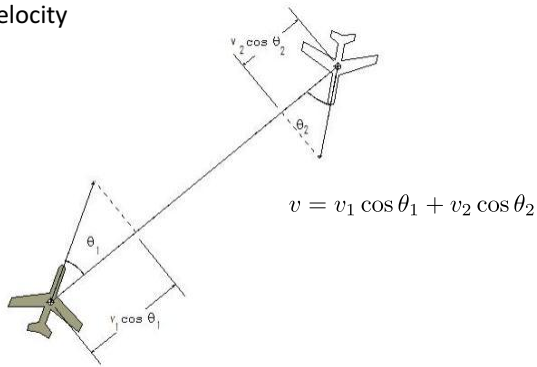
$$f_d = \frac{2v_r}{\lambda} = \frac{2v \cos \theta}{\lambda}$$

$$f_d = \frac{2v_r f_c}{c} \quad \therefore \frac{f_d}{f_c} = \frac{2v_r}{c}$$

- The Doppler shift is **directly proportional to the relative velocity** of target approaching or receding.

3.1 Doppler effect

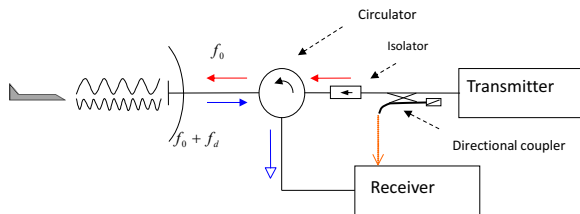
Relative velocity



Problem 3.1

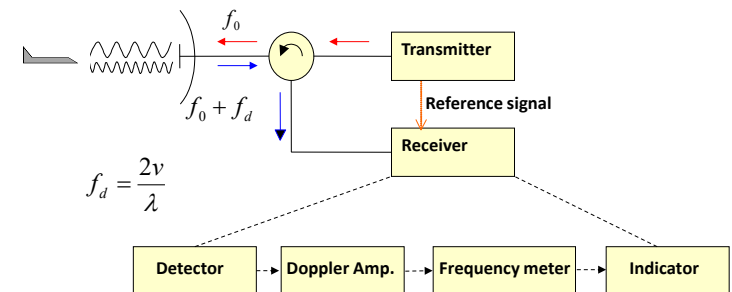
3.2 Continuous Wave Radar

Homodyne receiver



3.2 Continuous Wave Radar

Homodyne receiver



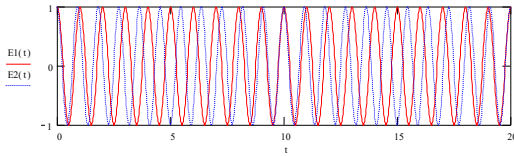
3.2 Continuous Wave Radar

Product detector and rectifier

Signals

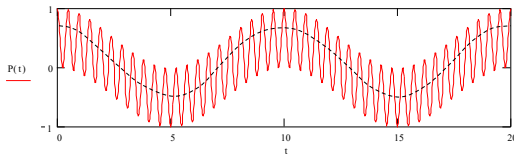
$$E_1(t) = A \cos(2\pi f_c t)$$

$$E_2(t) = A \cos(2\pi f_c t)$$



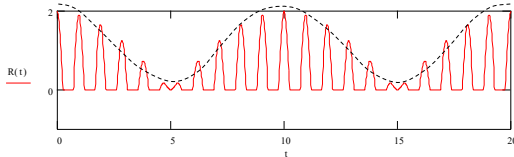
Product

$$P(t) = A_1 A_2 \cos(2\pi f_c t) \cos(2\pi f_c t)$$



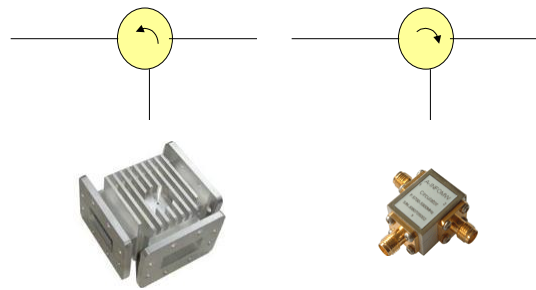
Rectification

$$R(t) = \begin{cases} E_1(t) + E_2(t) & \text{if } E_1(t) + E_2(t) \geq 0 \\ 0 & \text{if } E_1(t) + E_2(t) < 0 \end{cases}$$



3.2 Continuous Wave Radar

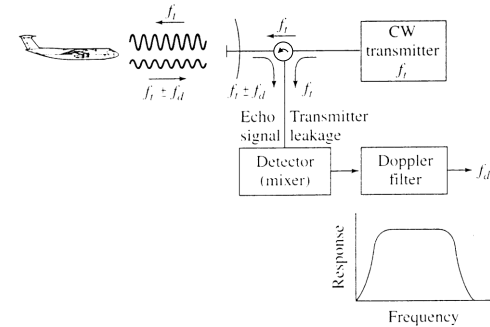
Circulators



Circulators can be used as duplexers

3.2 Continuous Wave Radar

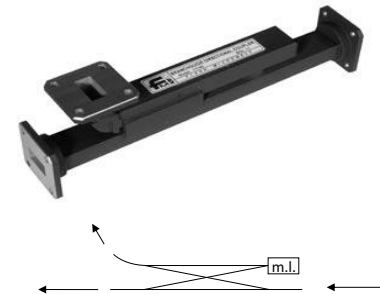
Homodyne receiver



The simplest schematic of a CW radar with homodyne detection is represented here. The same antenna is used for transmit and receive. Path are separated using a circulator. A leakage signal is used as reference signal

3.2 Continuous Wave Radar

Directional couplers



Directional coupler: Usually one arm (port) is ended by a matched load

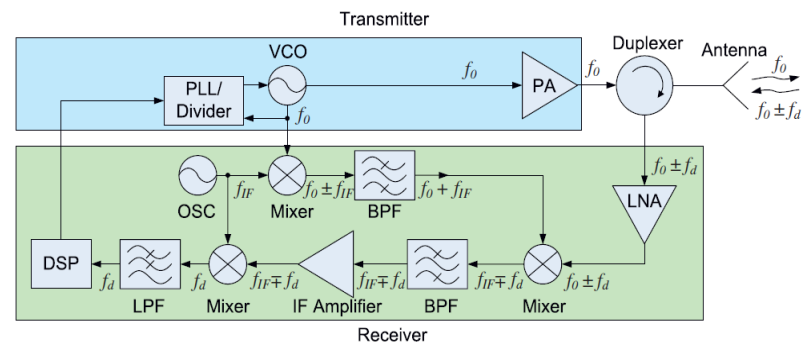
3.2 Continuous Wave Radar

Directional couplers



3.2 Continuous Wave Radar

Heterodyne receiver



3.2 Continuous Wave Radar

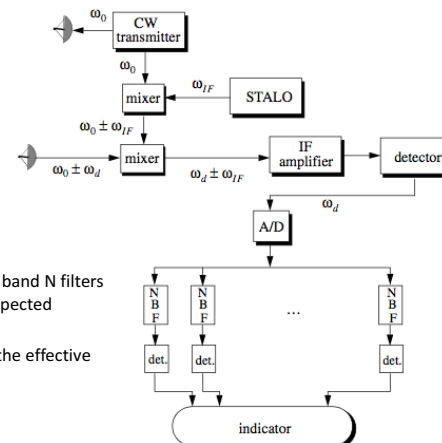
Heterodyne detection with separate antennas

For higher sensitivity an heterodyne detection and separate antennas should be used.



3.2 Continuous Wave Radar

Heterodyne detection with separate antennas



After A/D conversion narrow band N filters (NBF) follow, covering the expected Doppler frequency band;
If each NBF bandwidth is Δf , the effective Doppler bandwidth is $N\Delta f/2$

NBF – narrow band filters implemented using FFT

3.2 Continuous Wave Radar

Isolation between transmitter and receiver

In principle, a single antenna can be used, because there exists a frequency separation between the transmitted and received signal;

In practice there is always a leakage signal, that is directly injected from the receiver to the transmitter;

The leakage signal can be limited by two effects:

- The maximum power limit at the receiver;
- The maximum transmitter “humming” at the Doppler frequencies to be detected.

The best isolation is achieved using separate antennas.

3.2 Continuous Wave Radar

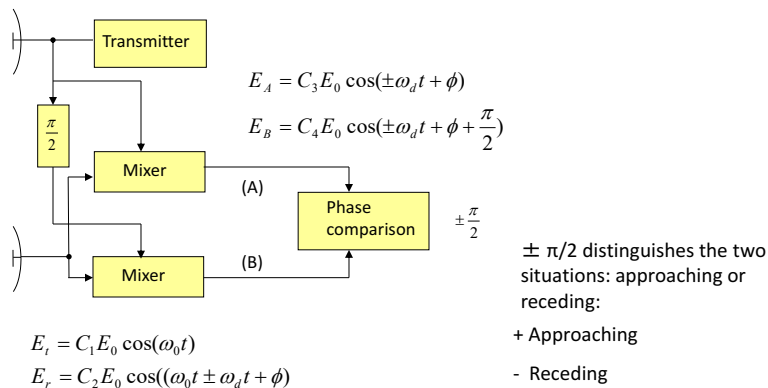
Doppler spectrum

The Doppler spectrum has a finite bandwidth due to effects such as:

- Finite illumination time $\sim 1/T_{il}$
- Target fluctuation components (ex: propellers) $\sim (50-100 \text{ Hz})$
- Radial acceleration $\sim (2a_r/\lambda)^{1/2}$

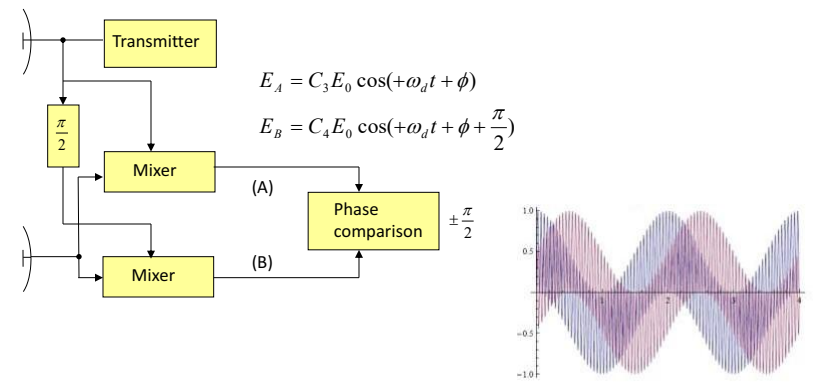
3.2 Continuous Wave Radar

Algebraic Doppler shift sign



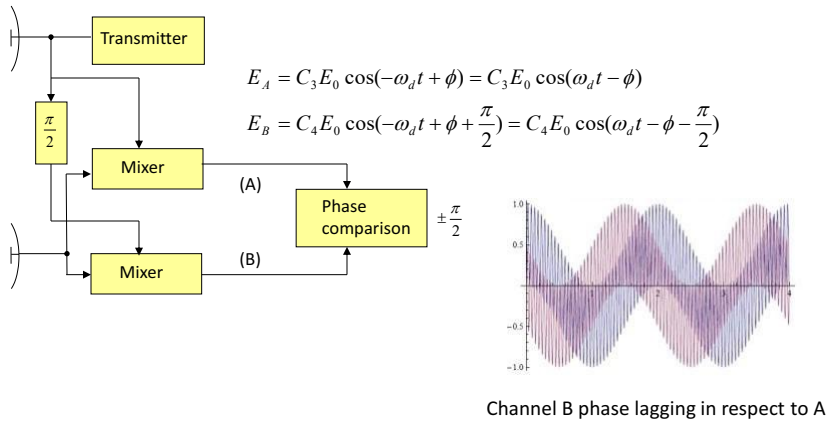
3.2 Continuous Wave Radar

Approaching target



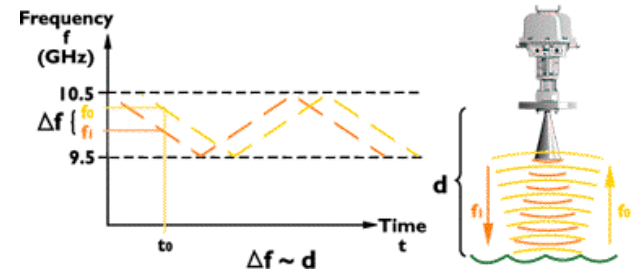
3.2 Continuous Wave Radar

Receding target



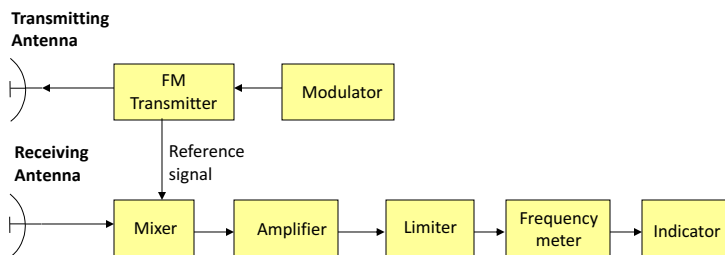
3.3 FMCW Radar

Distance measurement



3.3 FMCW Radar

Block diagram



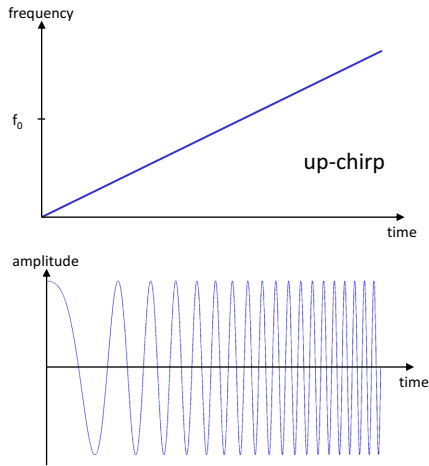
3.3 FMCW Radar

Principle of operation

- A periodic waveform, frequency modulating the transmitted signal, allows for the determination of the distance to a target, by comparing the modulation of the transmitted signal with the modulation of the returned signal.
- FMCW Radar is a radar transmitting a continuous carrier modulated by a periodic function such as a sinusoid or saw-tooth wave to provide range information.
- Modulation is the keyword, since this adds the ranging capability to FMCW radars with respect to unmodulated CW radars.

3.3 FMCW Radar

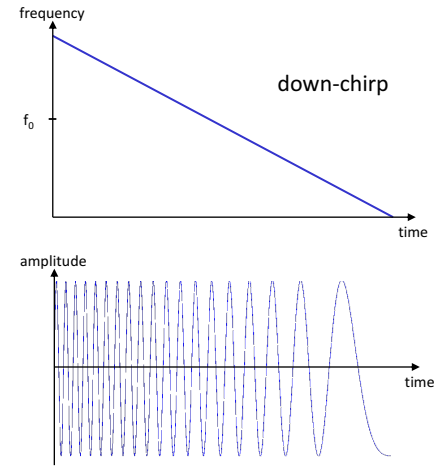
Linear FMCW (chirp)



Radar Systems – António Topa

3.3 FMCW Radar

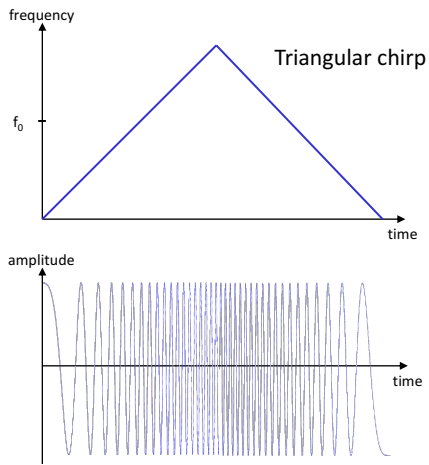
Linear FMCW (chirp)



Radar Systems – António Topa

3.3 FMCW Radar

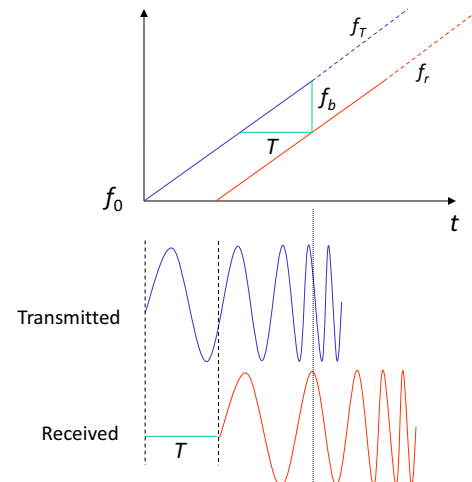
Linear FMCW (chirp)



Radar Systems – António Topa

3.3 FMCW Radar

FMCW Radar waveform



Beat frequency:

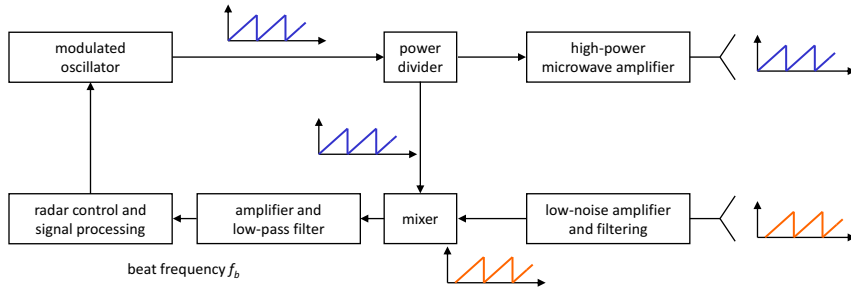
$$f_b = |f_T - f_r|$$

$$= \frac{2R}{c} \times \frac{\partial f_T}{\partial t}$$

Radar Systems – António Topa

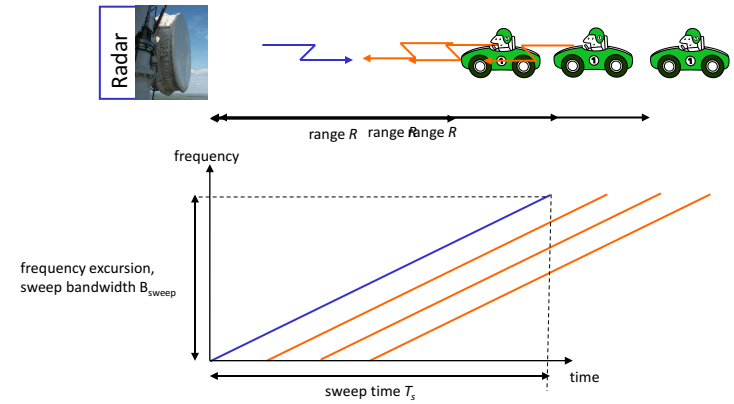
3.3 FMCW Radar

General block diagram



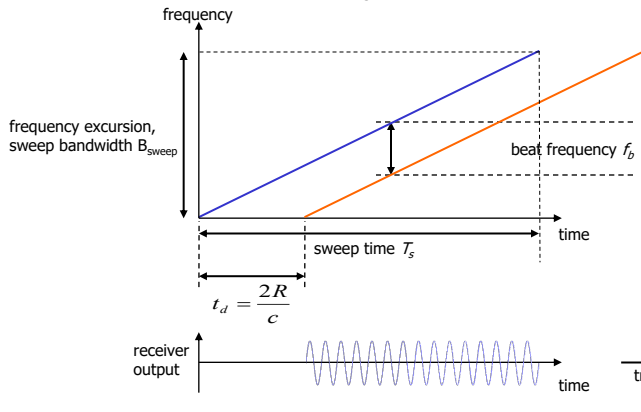
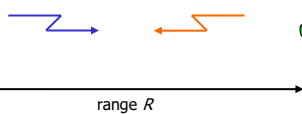
3.3 FMCW Radar

Static target



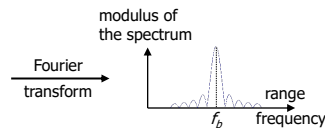
3.3 FMCW Radar

Static target



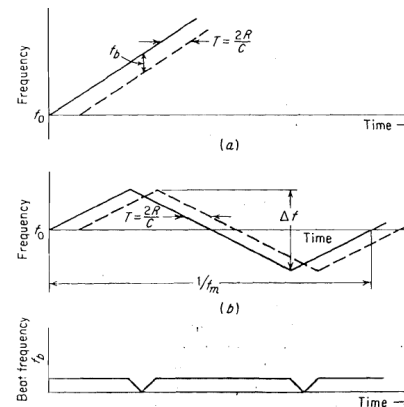
$$\frac{t_d}{T_s} = \frac{f_b}{B_{sweep}}$$

$$R = \frac{c T_s f_b}{2 B_{sweep}}$$



3.3 FMCW Radar

Triangular FMCW



The beat frequency is proportional to the target distance:

$$\frac{f_b}{T} = \frac{\Delta f}{T_m / 2} = 2 f_m \Delta f \quad T = \frac{2R}{c}$$

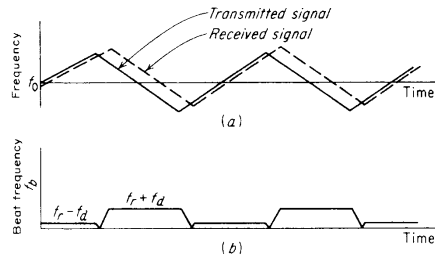
$$\therefore f_b = \frac{4R f_m \Delta f}{c}$$

The number N , of zero-crossings, per second (one per half a period $T_m/2$), is equal to f_b/f_m allowing the calculation of the distance to target:

$$R = \frac{cN}{4\Delta f}$$

3.3 FMCW Radar

Triangular FMCW with Doppler shift



f_r : Frequency shift due to modulation (stationary target)

f_d : Doppler shift

The half-periods of the modulation are affected by the Doppler shift, and as a result the beat frequencies are different:

$$f_{\uparrow} = f_r - f_d$$

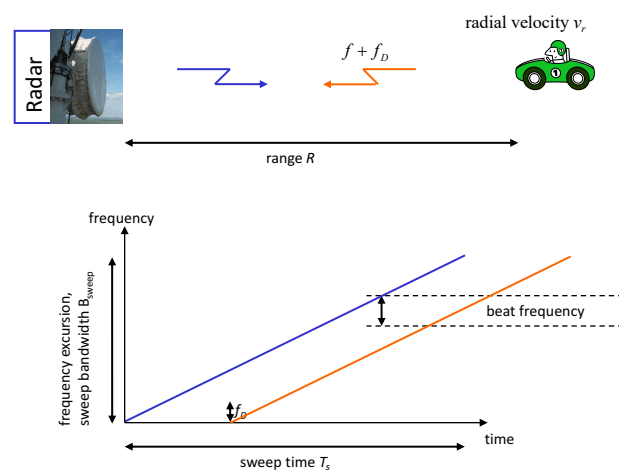
$$f_{\downarrow} = f_r + f_d$$

$$f_d = \left| \frac{f_{\uparrow} - f_{\downarrow}}{2} \right|$$

$$f_r = \frac{f_{\uparrow} + f_{\downarrow}}{2}$$

3.3 FMCW Radar

Moving target



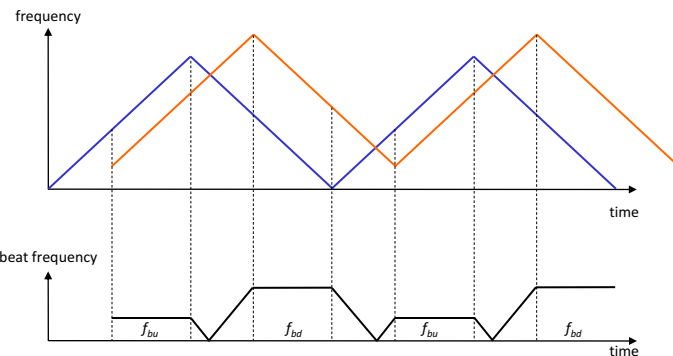
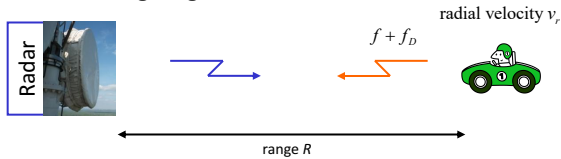
A moving target induces a Doppler shift

$$f_D = \frac{2v_r}{\lambda}$$

The beat frequency is not only related to the range of the target, but also to its relative radial velocity with respect to the radar.

3.3 FMCW Radar

Moving target



Beat frequency components due to range and Doppler frequency shift:

$$f_b = \frac{B_{sweep}}{T_s} \cdot \frac{2R}{c}$$

$$f_D = \frac{2v_r}{\lambda}$$

that are superimposed as

$$f_{bu} = f_b - f_d$$

$$f_{bd} = f_b + f_d$$

So, range and radial velocity can be obtained as

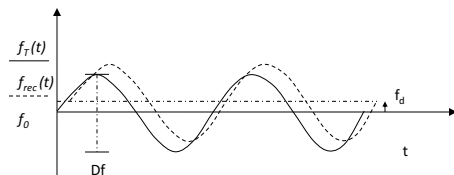
$$R = \frac{cT_s}{4B_{sweep}} (f_{bd} + f_{bu})$$

$$v_r = \frac{\lambda}{4} (f_{bd} - f_{bu})$$

Problem 3.5

3.4 Sinusoidal FMCW Radar

Instead of triangular modulation, sinusoidal modulation can be used.



It can be shown that the average beat-frequency over a modulation cycle is still proportional to distance.

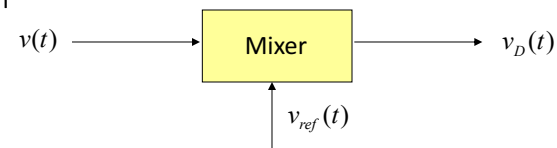
Representation of instantaneous frequencies of the transmitted and received signals:

$$f_T(t) = f_0 + \frac{\Delta f}{2} \sin(2\pi f_m t)$$

$$f_{rec}(t) = f_0 \pm f_d + \frac{\Delta f}{2} \sin\left(2\pi f_m \left(t - \frac{2R}{c}\right)\right)$$

3.4 Sinusoidal FMCW Radar

Detection



After filtering, retaining only the low frequency components:

$$v_D(t) = J_0(D) \cos(2\pi f_d t - \Phi_0) + 2J_1(D) \sin(2\pi f_d t - \Phi_0) \times \cos(2\pi f_m t - \Phi_m) - 2J_2(D) \sin(2\pi f_d t - \Phi_0) \times \cos[2(2\pi f_m t - \Phi_m)] + 2J_3(D) \sin(2\pi f_d t - \Phi_0) \times \cos[3(2\pi f_m t - \Phi_m)] - \dots$$

where: $D = \frac{\Delta f}{f_m} \sin(2\pi f_m R_0 / c)$ R_0 : Distance to target at $t = 0$

Φ_0 : Phase difference due to path $4\pi f_0 R_0 / c$

$\Phi_m = 2\pi f_m R_0 / c$

Problem 3.7

3.5 Multiple Frequency CW Radar

- Alternatively to frequency modulation CW radars can use multiple frequencies.
- When assuming that only transmission frequency is used the transmitted signal may be represented by

$$s(t) = A \sin(2\pi f_0 t)$$

- and the received signal by $s_r(t) = A_r \sin(2\pi f_0 t - \varphi)$ where $\varphi = 2\pi f_0 \frac{2R}{c}$

- Distance to target would then be $R = \frac{c\varphi}{4\pi f_0}$, $R = \frac{\lambda}{4\pi} \varphi$

- The maximum unambiguous range would correspond to $\varphi = 2\pi$, and would be $\lambda/2$ with no practical interest.

3.5 Multiple Frequency CW Radar

Double Frequency CW Radar

Assume now a signal composed by two frequencies

$$s_1(t) = A_1 \sin(2\pi f_1 t) \quad , \quad s_2(t) = A_2 \sin(2\pi f_2 t)$$

The received signal from a moving target would be represented by the sum of two components

$$s_{r1}(t) = A_{r1} \sin(2\pi f_1 t - \varphi_1) \quad , \quad s_{r2}(t) = A_{r2} \sin(2\pi f_2 t - \varphi_2)$$

where

$$\varphi_{1,2} = \frac{4\pi f_{1,2} R}{c}$$

After heterodyning, the phase difference between these signals would be

$$\Delta\varphi = \varphi_2 - \varphi_1 = \frac{4\pi R}{c} (f_2 - f_1) = \frac{4\pi R}{c} \Delta f$$

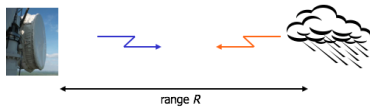
Distance is then obtained by $R = \frac{c\Delta\varphi}{4\pi\Delta f}$

The maximum unambiguous distance would then be $R_{MUR} = \frac{c}{2\Delta f}$

much higher than the one obtained with only one frequency

Problem 3.8

3.6 Atmospheric FMCW Radar



In an atmospheric FMCW radar the expected Doppler frequency shift usually has a small effect on the range extraction from the beat frequency, allowing a range estimation by comparing the phase of returns from successive sweeps.

The phase of the received signal is $\phi_r = \phi_t + \frac{4\pi R}{\lambda}$

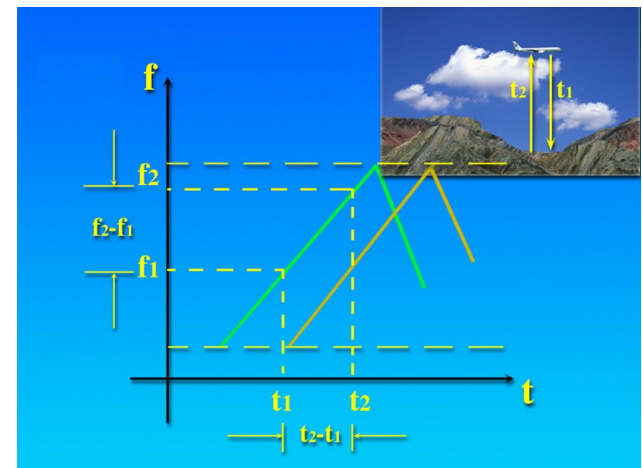
The change of the phase of the received signal with time is given by

$$\frac{d\phi_r}{dt} = \frac{4\pi}{\lambda} \frac{dR}{dt} = \frac{4\pi}{\lambda} v_r$$

The change of the phase of the received signal successive sweeps is given as

$$\frac{\Delta\phi_r}{T_s} = \frac{4\pi}{\lambda} v_r \quad \longrightarrow \quad v_r = \frac{\Delta\phi_r}{T_s} \cdot \frac{\lambda}{4\pi}$$

3.7 Altimeters



3.7 Altimeters



Band 4.2 - 4.4 GHz

Invented in 1924, by Espenschied

radar altimeter, radio altimeter, low range radio altimeter (LRRA) or simply RA

Used in commercial aviation for approaching and landing, specially under low visibility conditions

In civil applications, the reading is normally 2500 feet (760 m)

3.7 Altimeters



Aircrafts

- Landing and takeoff
 - Auto-Landing
 - Low Visibility
 - ILS (determination of Decision Height)
 - VTOL*
- AGL control **
 - Ground monitoring
 - Rescue mission with helicopters
- Ground Proximity Warning System

Satellites

- Oceans surface elevation observation
- Sea currents
- Ice layers height



* vertical takeoff and landing
** Airline ground landing

Error sources and precision

The radar altimeter precision is affected mainly by:

- Hydrometeors: cause refraction and attenuation
- Double reflection (“ Double Bounce”) and multi-path:
- Terrain irregularities or undesired reflections from objects that are not under the vertical relative to the aircraft

Present day precision found in the market

Model	Landing precision (m)
Miniature Radar Altimeter	0.5
Honeywell HG7808	0.6
FreeFlight TRA2000	1.5
FreeFlight Systems RA4000	0.6