

# Radar Systems

## Basic Values

Speed of light  $c = 300,000,000 \frac{m}{s}$

Boltzmann's constant  $K_B = 1.38 \times 10^{-23} \frac{J}{K}$

## Basic Formulas

### Geometry

Angle between vectors:

$$\theta = \arccos\left(\frac{a \cdot b}{|a| |b|}\right)$$

### Wavelength

$$\lambda = \frac{c}{f}$$

### Range / Time Relation

$$T_R = 2 \frac{R}{c}$$

$$R = \frac{c T_R}{2}$$

$1 \mu s = 150 m$

### Doppler Shift

$$f_D = \frac{2v \cos(\theta)}{\lambda}$$

### Pulse Repetition Frequency

$$PRF = \frac{1}{\tau}$$

### dB

$$P_{Watt} = 10^{\frac{P_{dbm} - 30}{10}}$$

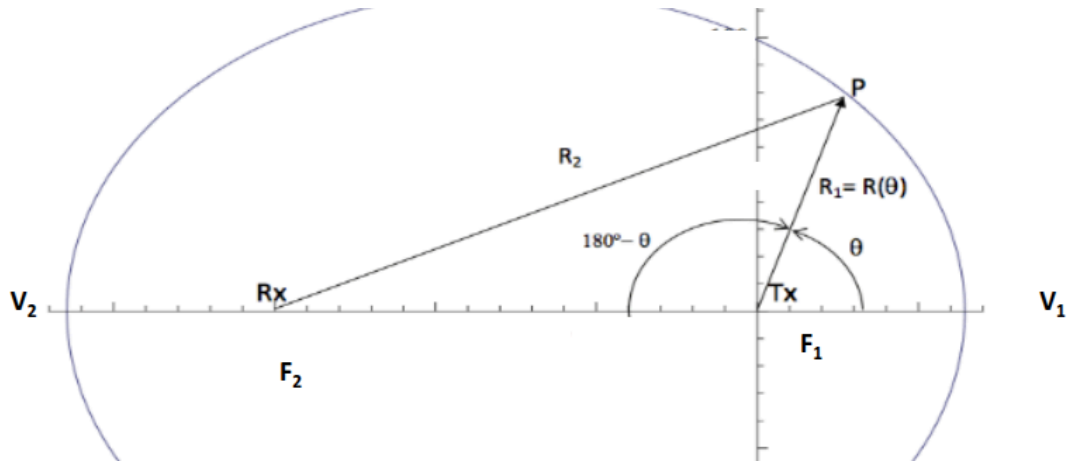
$$\text{ratio} = 10^{\frac{dB}{10}}$$

$$dB = 10 \log_{10}(\text{ratio})$$

### Maximum Unambiguous Range

$$MUR = \frac{c}{2 PRF}$$

## Ellipse Equations



$$e = \frac{\overline{F_1 F_2}}{\overline{V_1 V_2}}$$

$$a = \frac{\overline{V_1 V_2}}{2}$$

$$R_1(\theta) = a \frac{1 - e^2}{1 + e \cos(\theta)}$$

## Radar

### Radar Range

$$P_r = P_t \cdot \frac{\lambda^2 G^2 \sigma}{(4\pi)^3 R^4}$$

$$P_t = \frac{(4\pi)^3 R^4 P_r}{\lambda^2 G^2 \sigma}$$

$$R = \sqrt[4]{\left[ \frac{P_t \lambda^2 G^2 \sigma}{(4\pi)^3 P_r} \right]}$$

$P_r$  = Received Power

$\sigma$  = RCS (Radar Cross Section)

$\lambda$  = Wavelength

$G$  = Antenna Gain

$P_t$  = Transmitted Power

$R$  = Range

For a Bistatic Radar, replace  $R^4$  with  $R_t^2 R_r^2$  where  $R_t$  is the range from the transmitter to the target and  $R_r$  is the range from the target to the receiver.

### Minimum Target Separation

$$S = 2R \sin\left(\frac{\theta}{2}\right)$$

$\theta$  = Half Power Beamwidth

$R$  = Range

### Range Resolution

$$\Delta R = \frac{c\tau}{2}$$

$\tau$  = Pulse Duration

### Cell Volume

Cylinder of Radius  $\frac{S}{2}$  and length  $\Delta R$

$$V = \pi \left( \frac{S}{2} \right)^2 \Delta R$$

### SNR

$$T_{\text{eq}} = T_A + (F - 1)T_0$$

$$\text{SNR} = \frac{P_r}{K_B T_{\text{eq}} \Delta f}$$

$K_B$  = Boltzmann's constant =  $1.38 \times 10^{-23} \frac{J}{K}$

$F$  = Noise Figure

$T_A$  = Antenna Noise Temperature

$T_0$  = Reference Temperature

$\Delta f$  = Receiver Noise Bandwidth

SNR Value:

$P_D$	$P_{fa}$									
	$10^{-3}$	$10^{-4}$	$10^{-5}$	$10^{-6}$	$10^{-7}$	$10^{-8}$	$10^{-9}$	$10^{-10}$	$10^{-11}$	$10^{-12}$
.1	4.00	6.19	7.85	8.95	9.94	10.44	11.12	11.62	12.16	12.65
.2	5.57	7.35	8.75	9.81	10.50	11.19	11.87	12.31	12.85	13.25
.3	6.75	8.25	9.50	10.44	11.10	11.75	12.37	12.81	13.25	13.65
.4	7.87	8.85	10.18	10.87	11.56	12.18	12.75	13.25	13.65	14.00
.5	8.44	9.45	10.62	11.25	11.95	12.60	13.11	13.52	14.00	14.35
.6	8.75	9.95	11.00	11.75	12.37	12.88	13.50	13.87	14.25	14.62
.7	9.56	10.50	11.50	12.31	12.75	13.31	13.87	14.20	14.59	14.95
.8	10.18	11.12	12.05	12.62	13.25	13.75	14.25	14.55	14.87	15.25
.9	10.95	11.85	12.65	13.31	13.85	14.25	14.62	15.00	15.45	15.75
.95	11.50	12.40	13.12	13.65	14.25	14.64	15.10	15.45	15.75	16.12
.98	12.18	13.00	13.62	14.25	14.62	15.12	15.47	15.85	16.25	16.50
.99	12.62	13.37	14.05	14.50	15.00	15.38	15.75	16.12	16.47	16.75
.995	12.85	13.65	14.31	14.75	15.25	15.71	16.06	16.37	16.65	17.00
.998	13.31	14.05	14.62	15.06	15.53	16.05	16.37	16.7	16.89	17.25
.999	13.62	14.25	14.88	15.25	15.85	16.13	16.50	16.85	17.12	17.44
.9995	13.84	14.50	15.06	15.55	15.99	16.35	16.70	16.98	17.35	17.55
.9999	14.38	14.94	15.44	16.12	16.50	16.87	17.12	17.35	17.62	17.87

### False Alarm Calculation

$$P_{FA} = \frac{1}{B_{IF} T_{fa}}$$

$$P_{FA} = e^{-\frac{V_T^2}{2} \cdot \Psi}$$

$P_{FA}$  = Probability of False Alarm

$$V_T = \sqrt{2 \cdot \Psi \cdot \log_e \left( \frac{1}{P_{FA}} \right)} = n_{rms} \sqrt{\log_e \left( \frac{1}{P_{FA}} \right)}$$

$B_{IF}$  = Detector Frequency Bandwidth

$T_{fa}$  = Time Interval for False Alarm

$$n_{rms} = \sqrt{2 \cdot \Psi}$$

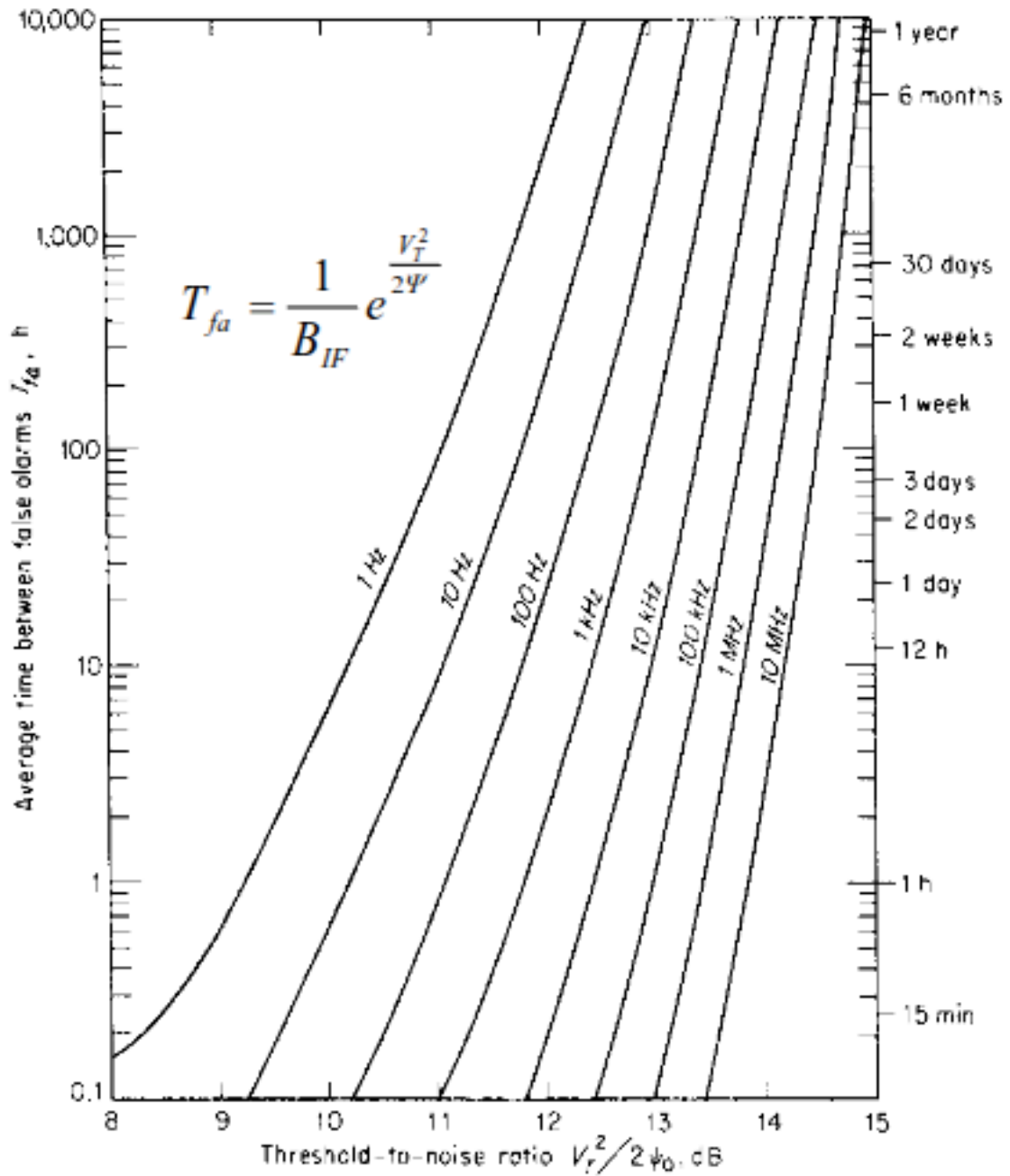
$$\Psi = \frac{n_{rms}^2}{2}$$

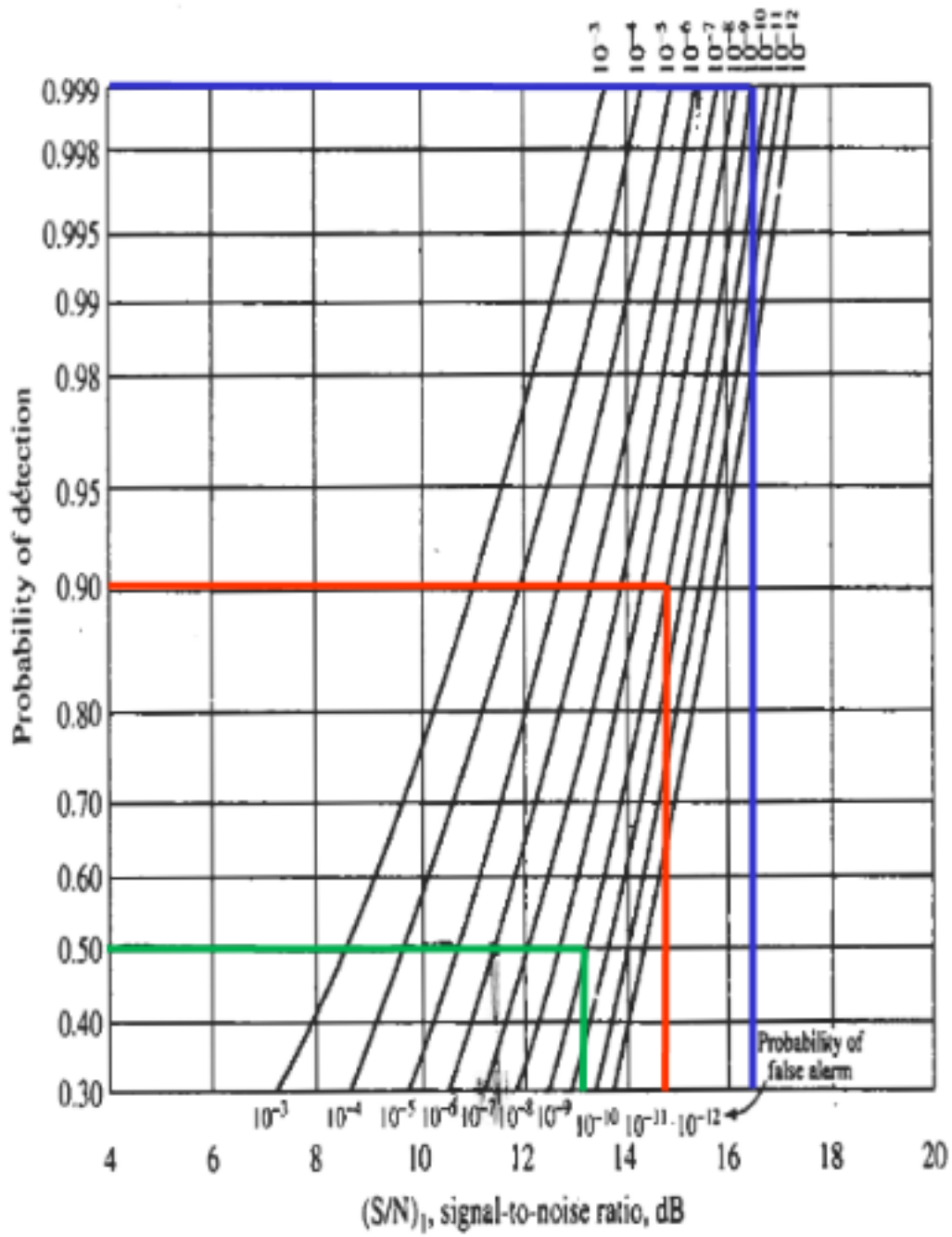
$V_T$  = Threshold Voltage

$$SNR_T = \frac{V_T^2}{2 \cdot \Psi}$$

$$\text{SNR} = \log_e \left( \frac{1}{P_{\text{FA}}} \right)$$

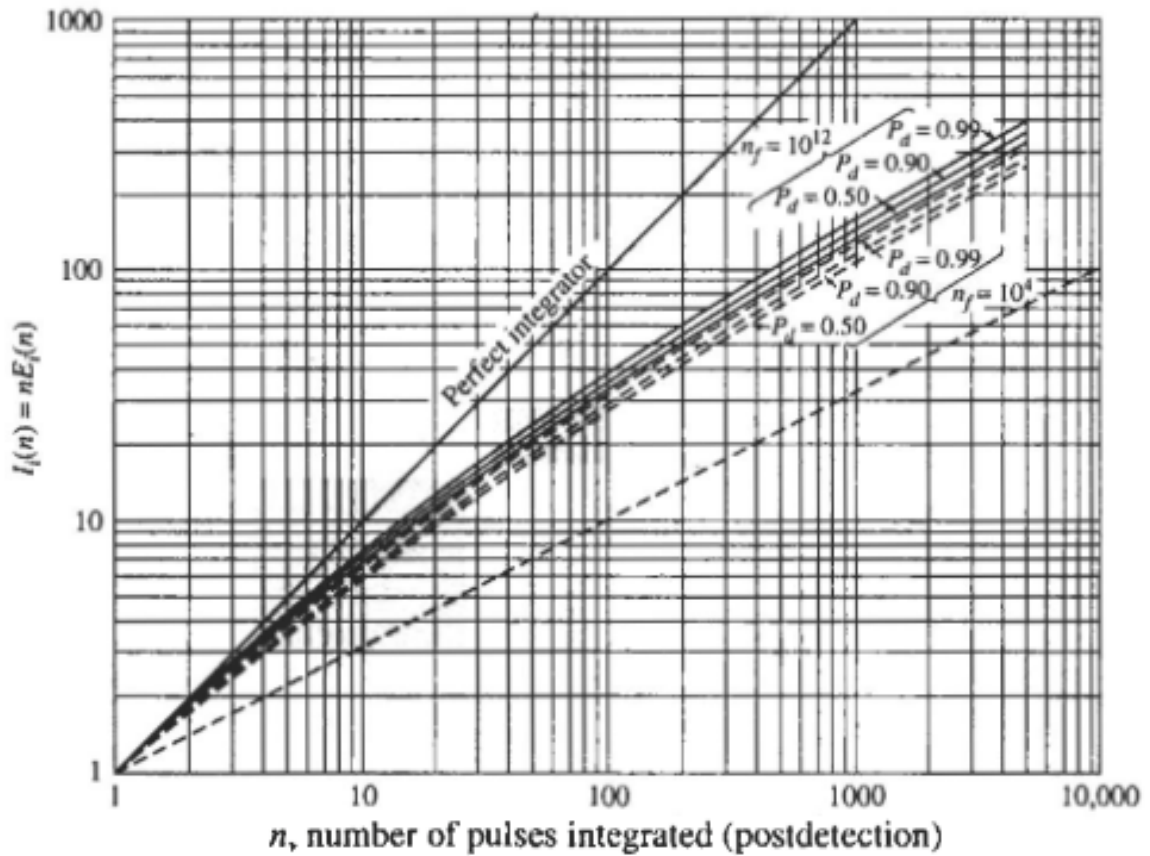
$$\left( \frac{S}{N} \right)_T = 10 \log_{10} \left( \log_e \left( \frac{1}{P_{\text{FA}}} \right) \right) \text{ dB}$$





### Integration

Efficiency of Integrator  $E_i(n) = \frac{I(n)}{n}$  Integration Loss  $L_i(n) = 10 \log\left(\frac{1}{E_i(n)}\right)$



## CW Radar

### Doppler Shift

Shifted frequency:

$$f' = f \frac{c + v}{c}$$

Frequency shift:

$$f_d = f' - f = \frac{fv}{c} = \frac{v}{\lambda}$$

For Monostatic Radar, replace  $v$  with  $2v \cos(\theta)$

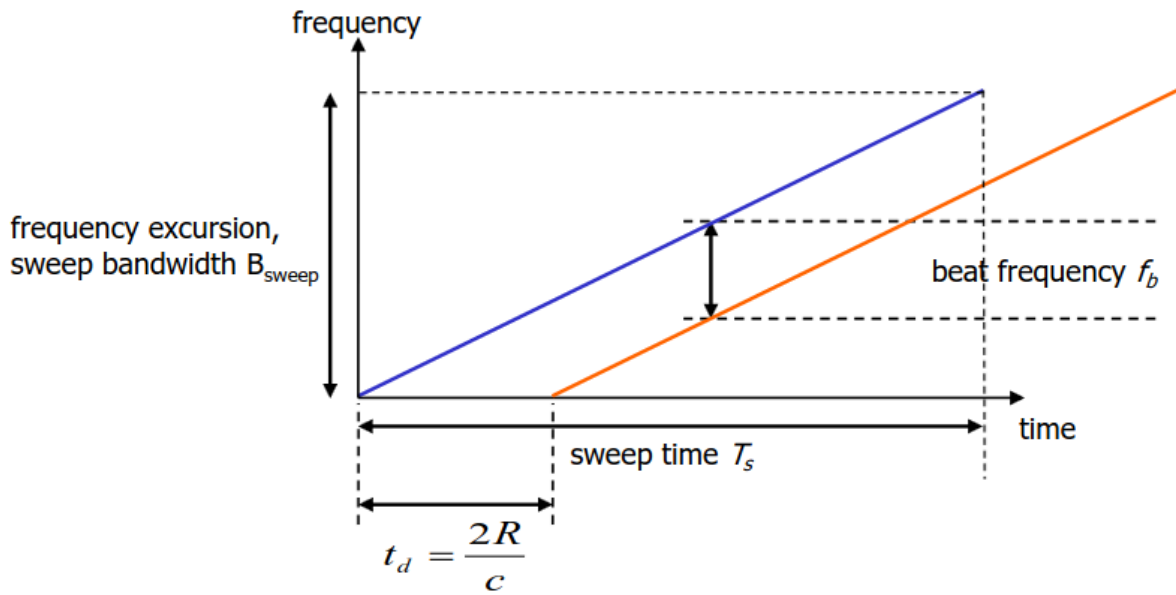
### FMCW Radar

$$\frac{t_d}{T_s} = \frac{f_b}{B_{\text{sweep}}}$$

$$R = \frac{cT_s f_b}{2B_{\text{sweep}}}$$

Zero Crossings:

$$R = \frac{cN}{4\Delta f}$$



$$\frac{t_d}{T_s} = \frac{f_b}{B_{\text{sweep}}}$$

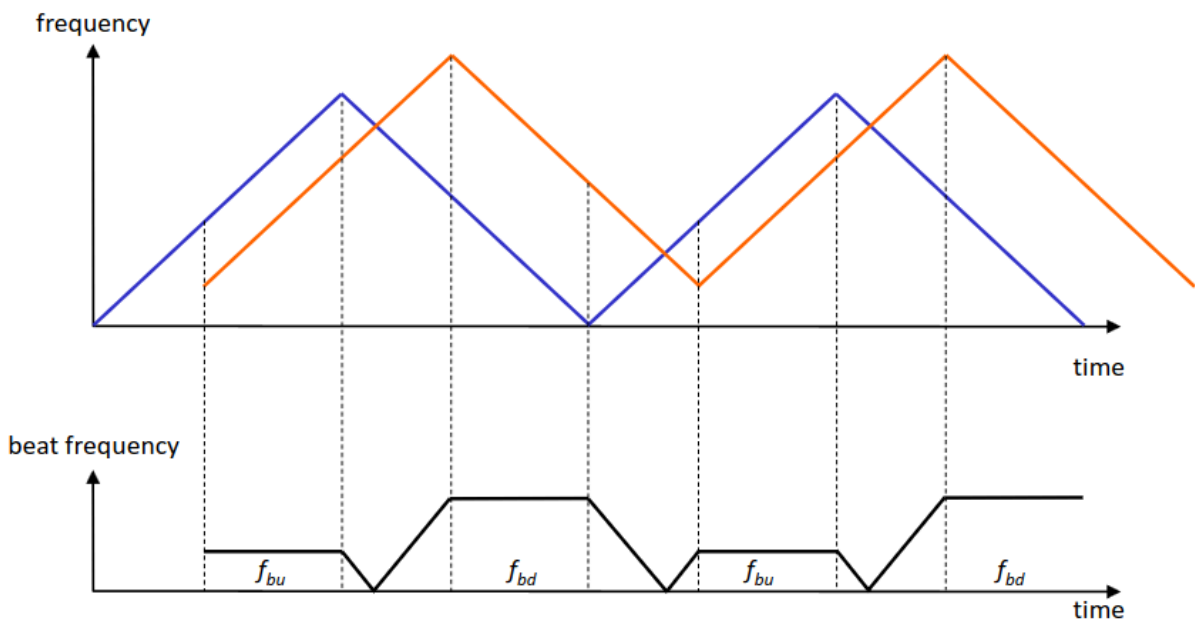
$$R = \frac{cT_s f_b}{2B_{\text{sweep}}}$$

Beat frequency:

$$f_b = \frac{4Rf_m \Delta f}{c}$$

$\Delta f$  = Sweep frequency range

With Doppler shift:



$$R = \frac{cT_s f_b}{4B_{\text{sweep}}} (f_{bd} + f_{bu})$$

$$v_r = \frac{\lambda}{4}(f_{bd} + f_{bu})$$

### MTI

nth blind speed:

$$v_{\text{blind},n} = n \frac{\lambda}{2} f_p$$

For staggered pulses:

$$v_{\text{blind}} = \text{average of pulse proportions} \cdot v_{\text{blind}_1}$$